The Limits to Partial Banking Unions: A Political Economy Approach

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Abstract

Will a banking union increase the welfare in all Eurozone countries? This paper studies the desirability of a banking union given political economy frictions in the member countries. Bank recapitalizations are carried out by self-interested policymakers who divert public funds towards socially inefficient rents. In equilibrium, a banking union increases recapitalizations, but it can also increase rent-seeking and decrease consumer welfare. I consider the effectiveness of electoral accountability and limits on public debt in increasing welfare. When used alone, neither policy can increase welfare for all countries. When used together, the policies are complementary, and a Pareto improvement is achieved.

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1 Introduction

The recent banking and sovereign debt crises have renewed interest in creating common cross-country rules for government interventions in the banking sector. This has been particularly relevant for the Eurozone, given the large cross-border spillovers from public bailouts. Naturally, the presence of such spillovers suggests that a banking union may deliver a Pareto improvement for all member countries. Domestic political economy constraints may, however, interfere with the functioning of such a supranational institution. First, policymakers may want to keep certain policy decisions under their direct control rather than delegating them to the supranational level. Second, once a banking union is in place, policymakers may divert resources towards socially inefficient rents. This raises the question of whether such a banking union can improve consumer welfare and achieve a more efficient supranational coordination of government interventions in the banking sector.

The policy debates surrounding the creation of a European Banking Union highlight how such issues pertaining to domestic political economy considerations affect the supranational coordination of banking policy. In drafting the plan for a banking union, the Eurozone countries have reached agreement on a unique supranational supervisory authority (the Single Supervisory Mechanism), in charge of coordinating bank supervision and regulation. Yet, another key component of the banking union is the Single Resolution Mechanism that would coordinate responses to a banking crisis. Recent proposals for this mechanism leave national authorities in each country with significant decision powers over the management of a banking crisis and the funding of bailouts.\footnote{A summary of these proposals and the progress towards achieving them is provided in European Commision (2012) and European Commission (2014). State sovereignty over decisions that affect national banks, voter concerns over the use of public funds towards bank bailouts, and domestic political rent-seeking have been indicated as major factors driving the banking union structure (see Constancio 2014).} Such a system in which a banking union falls short of a fully centralized mechanism is referred to as a ‘partial banking union.’ This paper considers a supranational arrangement in the form of a partial banking union and studies
the welfare effects caused by domestic political economy distortions.

The case of the Spanish savings and loan sector (the ‘cajas’) provides an illustration of the type of political economy distortion analyzed in this paper and of the role it played in the recent banking crisis.\(^2\) The Spanish cajas were led by politically appointed executives, and the political pressures faced by these executives affected the types of loans that they extended in the pre-crisis period. For example, regional governments used the cajas to fund projects that had little social benefit, but served political interests (e.g., airports with no flights, unused theme parks).\(^3\) During the crisis, local policymakers decided to rescue undercapitalized cajas by merging them, and these mergers were based on political and regional motives rather than economic efficiency. These inefficient mergers led to the creation of larger troubled entities, increasing the cost of public bailouts and the pressure on public finances in Spain and in the Eurozone. Some of the public funds that were used to recapitalize the troubled cajas covered losses from large, unregulated payments taken by politically appointed board members just prior to the government intervention.\(^4\)

The above features of a partial banking union and of domestic politics are built into a model that sheds light on the welfare effects of such a supranational arrangement, in which financial integration is not accompanied by political integration. I model a union of governments that are electorally accountable to voters within their own country and that have policy objectives that may differ from their voters’. Each government can provide public funds to distressed banks in its country through a process of bank recapitalizations, and these policies have cross-country spillover effects. The spillovers provide a motivation for cross-country transfers and for the supranational coordination of recapitalization policies. The model considers such an arrangement in the form of a partial banking union, in which rules for recapitalizations and cross-country transfers are determined at the supranational level, but each country’s government can decide how to allocate recapitalization funds,

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\(^2\)Discussed in greater detail in Garicano (2012) and Cuñat & Garicano (2009).
\(^4\)Ibid.
within the constraints of the supranational rules. To simplify, I consider the case in which all banks that need recapitalizations are located in one country, and the allocation of recapitalization funds in that country is decided by a self-interested policymaker. This policymaker faces a trade-off regarding the use of public funds, as the public budget can also be used for political rents and non-financial public goods (e.g., infrastructure projects). The paper focuses on Markov Perfect Equilibria, when policies are decided every period, and agents lack commitment power.

The paper shows that creating a partial banking union when policymakers are self-interested can reduce consumer welfare in the country that receives supranational transfers. This happens because the partial banking union gives policymakers incentives to increase rent-seeking. If the country receiving transfers carries a small weight in the partial banking union relative to the country giving transfers, the contract between the two countries keeps the rent-seeking policymaker in the receiving country indifferent to participating in the banking union. This implies that the country receiving transfers is required to increase government spending on recapitalizations. Yet, the required increase in recapitalizations also induces an increase in rent-seeking. The result is a decrease in the provision of domestic public goods, and this leads to lower welfare in the receiving country. This result highlights how the lack of political integration can undermine the benefits of financial integration when supranational authorities cannot restrict rent-seeking.

The political economy distortions exist only in the receiving country, so a first approach to correct these distortions could be to improve the domestic institutions of electoral accountability in this country. Electoral accountability refers to voters’ ability to remove elected politicians from office. As in the models developed by Barro (1973) and Ferejohn (1986).
policy integration, weak electoral or institutional control over politicians in the
peripheral Euro countries has been indicated as one reason why cross-country
transfers are difficult to achieve. These results, however, suggest that better
electoral accountability of politicians in the receiving country may not also
lead to higher welfare in the donor country, even if it decreases rent-seeking in
the receiving country.

Another possible resolution to the political economy friction is to restrict
the receiving country’s ability to borrow, through supranational fiscal rules
that constrain debt accumulation. Such fiscal rules have the effect of reducing
both overall spending and rents. The reason for this is that fiscal rules alone
cannot restrict the spending on rents without also restricting spending in gen-
eral. This results in both insufficient recapitalizations and insufficient public
good provision in the country receiving transfers. Although fiscal rules are
beneficial for the donor country, consumer welfare decreases in the receiving
country.

While neither policy alone can achieve a Pareto improvement, the model
shows that such an improvement can be obtained if these policies are imple-
mented together, optimized for each other. Electoral accountability constrains
the policymaker to reduce rent-seeking without decreasing spending on pub-
lic goods and recapitalizations. Fiscal rules ensure that the higher spending
on public goods is not done through increases in debt, but rather through
larger decreases in rent-seeking. The outcome is that higher recapitalizations
are achieved, while rent-seeking is controlled. Therefore, these two policies
together can deliver a Pareto improvement over the case without a banking
union.

The above results highlight how policies aimed at tackling one source of
inefficiency can have negative welfare implications by augmenting other in-
ceptive problems. Moreover, domestic political economy distortions may have
welfare implications at the supranational level, and they may make it costlier
to implement cross-country policies. This seems particularly relevant for the
Eurozone, where banking policies are only partially centralized, and signifi-
cant decision power still lies at the level of each country’s government. The
results suggest that supranational policies must be complemented by domestic policies in order to overcome the spillovers generated by domestic political economy distortions.

**Related Literature.** The interplay between fiscal policy and financial integration has been vastly studied in the literature. Yet, the main focus for most of the work in this area has been on optimal policy design with a benevolent government. This includes the study of optimal fiscal policy coordination (Kehoe 1987, Chari & Kehoe 1990, Beetsma & Lans Bovenberg 1998), fiscal rules in currency unions (Von Hagen & Eichengreen 1996, Ferrero 2009), or the role of fiscal transfers in providing efficient insurance within a currency union (Farhi & Werning 2012). All these papers abstract from the effects of political economy distortions or political decision-making. By contrast, this paper considers the issue of policy coordination with financial integration, taking into account the political economy issues that emerge where there is no political integration and policymakers are self-interested. Therefore, this paper is most closely related to the political economy work that considers the effects of different political institutions in the context of fiscal or financial integration (Tabellini 1990, Lohmann 1993, Persson & Tabellini 1996a, Persson & Tabellini 1996b). Whereas this literature focuses mainly on the effects of different electoral institutions and the aggregation of voter preferences, this paper considers the issue of political rent-seeking and examines the distortion to supranational policies due to domestic rent-seeking and imperfect electoral accountability.

The link between financial integration and domestic public debt in the presence of political economy constraints has also been studied by Tabellini (1990) and Azzimonti, de Francisco & Quadrini (2014), who show how fiscal or financial integration can lead to higher public debt due to political economy biases. This paper, however, highlights a different channel for the increase in public debt. Debt does not increase due to lower costs of borrowing (as in Tabellini 1990) or the aggregation of heterogeneous voter preferences (as in Azzimonti, de Francisco & Quadrini 2014), but rather because cross-country
transfers create higher incentives for current spending and rent-seeking. The increase in debt is directly linked to the existence of supranational agreements in the absence of political integration. In a set of papers also motivated by the European supranational institutions, Persson & Tabellini (1996a) and Persson & Tabellini (1996b) study cross-country insurance and the effect of fiscal transfers on welfare under different political decision-making institutions, specifically direct voting versus bargaining. This paper provides a complement to their results. While their papers highlight the inefficiencies that emerge under various institutions of collective choice – voting versus bargaining, this paper considers inefficiencies rooted in domestic institutions – rent-seeking and imperfect electoral control. Moreover, it presents another channel through which domestic institutions affect supranational agreements: that of rule implementation (the allocation of transfers by the local policymaker) rather than rule selection (the collective choice of transfers).

The modeling approach in this paper uses a principal-agent framework similar to those developed in Acemoglu (2005) and Acemoglu & Robinson (2006), which feature stochastic politician replacement costs, and in Yared (2010), which models electoral incentives as voters’ demand for a minimal utility level each period. The model also builds on the framework developed in Acemoglu, Golosov & Tsyvinski (2008) and Acemoglu, Golosov & Tsyvinski (2011) but differs from these models in two main ways. First, it focuses on Markov Perfect Equilibria as opposed to the best Subgame Perfect Equilibrium; and second, it considers an endowment economy without capital, but with public debt, supranational transfers and limits on spending and debt. Finally, this model links rent-seeking to recapitalizations using an approach similar to that of Milesi-Ferretti (2004), which models ‘creative accounting’ as the difference between a true fiscal variable and its corresponding ‘measured’ variable, then uses this difference over the business cycle to infer the effect of budget rules on fiscal policy. In this paper, supranational rules are imposed on spending measures which can differ from the true spending on recapitalizations, due to the presence of rents. The desirability of supranational controls over domestic spending has also been examined in Dewatripont & Seabright (2006), but in
the context of a politician whose type is unknown to voters, and who uses domestic spending to signal his type. This paper considers the role of supranational controls in a model without private information, where the politician has a direct preference for rent-seeking.

The rest of the paper is organized as follows. Section 2 presents the problem in a two-period model and illustrates the main results of the model. Section 3 gives the setup of the dynamic model. Section 4 presents the analysis of the model and the welfare effects of a partial banking union in the dynamic model. Section 5 analyzes the effects of higher electoral accountability and those of fiscal rules. Section 6 concludes, and the Appendix contains the proofs.

2 A Two-Period Model

This section presents a two-period model that illustrates the main results of the paper. It highlights the different driving forces of the model and the intuition behind the results. In Section 3, the model is extended to a dynamic framework, which shows that the results continue to hold in a more general setting, even under a weaker condition on electoral accountability. Moreover, it illustrates the effects of public debt on the sharing of recapitalization costs between countries.

Consider a two-period economy, with $t = 0, 1$. The economy consists of two countries, a donor country and a home country, and a supranational authority which plays the role of a Principal who proposes the terms of a partial banking union between countries. Each of the two countries is made up of a continuum of mass 1 of identical households.

2.1 Households

At date 0, all households start with a perfectly diversified portfolio of risky projects, in the form of deposits in banks. Home households hold deposits $\omega^H$, and donor (foreign) households hold deposits $\omega^F$. The assumption of different

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6 A more detailed description of the banks is provided in the Appendix.
sizes for the deposits is made because this difference determines the size of the cross-country spillovers. The risky projects are owned by banks located in the home country, and the projects pay off at the end of period 0. At the beginning of period 0, an aggregate shock $\theta \in (0, 1)$ is realized and observed by all agents. Following the shock, a fraction $\theta$ of the project portfolio becomes distressed, and it pays off 0 unless additional funds $x$ are reinvested, up to the original investment level ($x \leq \theta(\omega^H + \omega^F)$).\footnote{The liquidity shock is modeled as a simplified version of the one in Holmström & Tirole (1998).} I assume that banks have no access to a private borrowing market, so that reinvestment funds can only be provided by the government through public recapitalizations. Moreover, the reinvestment funds $x$ can be supplied to banks by their own government only; the donor government cannot directly recapitalize banks in a foreign country. The liquidity shock therefore motivates the need for government intervention in this model. A key assumption is that reinvestment funds cannot be targeted, so both the home and donor households benefit from the reinvestment. This benefit is proportional to each country’s share of deposits, where I denote by $\sigma \equiv \frac{\omega^H}{\omega^H + \omega^F}$ the share of deposits held by the home country households. At the end of the period, the projects that continue after the shock yield a rate of return of $R > 1$.

In the second period, all households hold safe deposits in banks, with values $\omega^H$ for the home households and $\omega^F$ for the donor households, and rate of return of 1. The assumption of a second period without aggregate shocks is made for simplicity. It creates a role for public debt in smoothing public good provision over time, as further shown below.

Each period, households derive utility from private consumption equal to their deposit returns. They also derive utility from a domestic public good $g^H$ provided by the government. Their preferences are given by\footnote{For ease of notation, I omit the subscripts for the period 0 variables and keep only the subscripts for the period 1 policies.}

$$U^j(x, g^j, g^j_1) = u(R(1 - \theta)\omega^j + Rx^j) + w(g^j) + \beta (u(\omega^j) + w(g^j_1)),$$
where \( j = H, F \), \( x^H = \sigma x \), \( x^F = (1 - \sigma)x \); \( \beta \in (0, 1) \) is the social discount rate and the inverse gross interest rate, and \( u(\cdot) \) and \( w(\cdot) \) are strictly concave, increasing, \( 0 < u'(0) < \infty, 0 < w'(0) < \infty, \lim_{g \to \infty} w'(g) = 0 \). Notice that both home and donor household utilities depend on the recapitalization \( x \) decided by the home government.

## 2.2 Donor Government

The donor country government is assumed to be free of any political economy distortions, so that its preferences are identical to those of households in that country. The government maximizes

\[
U^F(x, g^F, g_1^F) = u(R(1 - \theta)\omega^F + R(1 - \sigma)x) + w(g^F) + \beta \left( u(\omega^F) + w(g_1^F) \right).
\]

Each period, the donor government receives an endowment \( e^F \). With this endowment, it can finance the domestic public good, and it can make transfers \( \tau \) to the supranational authority at date 0. The donor government does not have access to any storage technologies and cannot borrow or lend against the future. I make this assumption for simplicity, to limit the role of the donor government to only that of providing transfers. The donor government’s budget constraint at dates 0 and 1 is given by \( g^F + \tau \leq e^F \) and \( g_1^F \leq e^F \).

## 2.3 Home Government

In the home country, government policy is decided by a self-interested politician, who maximizes a weighted sum of own utility from rents and household utility:

\[
V^H(r, x, g^H, g_1^H) = (1 - \gamma)v(r) + \gamma U^H(x, g^H, g_1^H),
\]

where \( v(\cdot) \) is weakly concave and increasing, \( v' < \infty \), and \( \gamma \in (0, 1) \) represents the weight placed on household utility relative to rents.

The home government receives an endowment \( e^H \) each period and can take on one-period debt \( b_1 \) in period 0, at rate \( \frac{1}{\beta} \), with an exogenous lower limit.
\( b \leq 0 \) and upper limit \( \bar{b} < e^H \). Assume period 0 starts with outstanding debt \( b = 0 \). The home government can also become part of a partial banking union in the current period. This involves receiving the transfer \( \tau \) from the supranational authority at date 0. In exchange, the government commits to an intervention level of \( \underline{x} \) towards bank recapitalizations. However, the spending on rents versus recapitalizations cannot be separately observed and verified by the supranational authority. Therefore, the intervention level \( \underline{x} \) can encompass both rents and recapitalizations; the required intervention level is satisfied as long as

\[
x + r \geq \underline{x}. 
\] (2)

This constraint will never be slack, since the supranational authority will never prefer to set an intervention level below what the politician would choose in the absence of this required level. Such a choice will decrease recapitalizations, since the politician will always choose to balance the increase in \( x \) and \( r \). This result emerges because the politician’s utility is concave in both rents and recapitalizations, so any incentive to increase recapitalizations will also give the politician the incentive to increase rents. The only way for the supranational authority to increase recapitalizations is to increase the required intervention level beyond what the politician would prefer, and to accept an increase in both rents and recapitalizations.

The constraints for the home government are:

\[
\begin{align*}
    r + x + g^H & \leq e^H + \beta_1 + \tau, 
    \quad (3a) \\
    r + x & \geq \underline{x}, 
    \quad (3b) \\
    b_1 & \leq \bar{b}, 
    \quad (3c) \\
    g^H_1 & \leq e^H - b_1. 
    \quad (3d)
\end{align*}
\]

Since rents are discussed in relation to recapitalizations, they are assumed away in the second period. The dynamic model presented in the next section considers the case of future rents and discusses the implications of changes in debt on expected rent-seeking.
Rent-seeking Process. This reduced-form relationship between rents and recapitalizations can be motivated by the following rent-extraction process, in the style of Coate & Morris (1995). The government can choose the degree of efficiency of its intervention in projects. The most socially efficient intervention provides reinvestment funds $x$ for the distressed projects. The politician can also choose less efficient interventions. In this type of interventions, he provides reinvestment funds $x$ but can also decide to expand the capacity of the project. Only the original project returns rate $R$, while the expansion of the project has a rate of return of 1. Moreover, the proceeds from the expanded project go to the politician, in the form of political rents. A politician who values rents more will choose to engage in a more socially inefficient intervention scheme, in order to increase rents. The total intervention will be equal to $x + r$, but only $x$ will constitute true recapitalizations. This rent-extraction mechanism can be used to model the example of the Spanish cajas mentioned in the introduction. The inefficient projects and payments made as a consequence of political rent-seeking can be modeled as expanding productive projects with extensions without added social value.

2.4 Partial Banking Unions

A transfer $\tau \geq 0$ and a level of intervention $\underline{x} \geq 0$ are set by the supranational authority to maximize a weighted sum of home and donor household utilities, with weight $\eta$ on home households:

$$\max_{\tau \underline{x}} \{ \eta U_H(x, g^H, g^H_1) + (1 - \eta) U_F(x, g^F, g^F_1) \}. \quad (4)$$

A partial banking union requires the participation of both the home and donor governments. The donor government must agree to make the transfer $\tau$, and the home government must agree to implement the required intervention level in exchange for the transfer. Neither government can commit to participating in the union. Therefore, it is necessary that each government finds the
banking union agreement to be preferable to autarky:

\[
U^F(x^s, g^F, g^F_1) \geq U^F(x^0, g^F_0, g^F_1), \quad (5)
\]

\[
(1 - \gamma)v(r^s) + \gamma U^H(x^s, g^H, g^H_1) \geq (1 - \gamma)v(r^0) + \gamma U^H(x^0, g^H_0, g^H_1), \quad (6)
\]

where \( \{r^s, x^s, g^H, g^H_1\} \) are policy choices made under the agreement \((\tau, x)\), and \( \{r^0, x^0, g^H_0, g^H_1\} \) are policy choices made without a banking union.

### 2.5 Timing

The timing of the model is as follows. In period 0, the supranational authority proposes a transfer \(\tau\) and intervention level \(\bar{x}\). The donor government decides whether to accept the proposed agreement, and make transfer \(\tau\), and the home government decides whether to accept the transfer in exchange for providing total intervention \(\bar{x}\). Finally, recapitalizations \(x\), rents \(r\), the domestic public good \(g^H\), and debt \(b_1\) are decided by the home government. In the second period, the governments provide the domestic public good, given the available budget after any debt repayments, and households consume the returns from second period deposits.

### 2.6 Recapitalization Choices

To shorten notation in the rest of the analysis, define \(u^H(x, \theta) \equiv u(R(1 - \theta)\omega^H + \sigma Rx)\), and define \(u^F(x, \theta)\) analogously for the donor government.

**Assumption 1** The following conditions are satisfied:

\[
u^H(0, \theta) > w'(0), \quad u^H(0, \theta) > v'(e^H),
\]

\[
u^H(0, \theta) < w' \left( e^H + \beta b - \theta \left( \omega^H + \omega^F \right) \right),
\]

where \(u^H\) and \(w'\) denote the first derivatives with respect to \(x\) and \(g^H\).

Assumption 1 states that in autarky the solution to the politician’s problem is interior with respect to recapitalizations. The politician chooses \(\{r, x, g^H, b_1\}\).
to maximize (1) subject to (3a)-(3d). The first-order conditions to the politician’s problem yield $(1 - \gamma)v'(r) = \sigma R\gamma u^H_r(x, \theta)$. This implies that any increase in recapitalizations will be accompanied by an increase in rents.

Given the policy choices made by each government, the supranational authority chooses a transfer $\tau$ and an intervention level $x$. The supranational authority can use the intervention level to increase the level of recapitalizations $x$ at the expense of lower domestic public good in the donor country or in the home country. It can use the transfer to provide the home government with more resources, the use of which is decided by the politician given the constraints described above.

**Assumption 2** The donor government’s endowment $e^F$ is sufficiently large so that $\eta u^H_r(\theta(\omega^H + \omega^F), \theta) + (1 - \eta)u^F_r(\theta(\omega^H + \omega^F), \theta) > (1 - \eta)w'(e^F)$ and $\eta u^H_r(\theta(\omega^H + \omega^F), \theta) + (1 - \eta)u^F_r(\theta(\omega^H + \omega^F), \theta) > (1 - \eta)w'(e^F - \theta(\omega^H + \omega^F) + x^0)$,

where $u^H_r$ and $u^F_r$ denote the first derivative with respect to $x$, $w'$ denotes the first derivative with respect to $e^F$, and $x^0$ denotes the autarky level of recapitalizations derived from maximizing (1) subject to constraints (3a) with $\tau = 0$, (3c) and (3d).

Under Assumption 2, the supranational authority prefers to offer positive transfers, because the cost of transfers – less public good in the donor country – is lower than the benefit of more recapitalizations. The second condition ensures that, if the supranational authority could directly recapitalize banks in the home country, then it would fully recapitalize these banks. This provides a benchmark from which we can analyze the distortions introduced by the political economy frictions. Yet, notice that the second condition of Assumption 2 does not mean that the supranational authority would necessarily prefer to recapitalize banks through transfers alone. The supranational authority has two instruments at its disposal, so it could also force an increase in recapitalizations by using the required intervention $x$. If the transfer $\tau$ is lower than the increase in interventions (recapitalizations and rents) required by $x$, then
part of the intervention has to be financed through higher public debt and a
decrease in the home country public good. The existence of these two sources
of funding raises two important questions: first, whether the supranational
authority will prefer to partially finance recapitalizations through increases
home country public debt; second, whether this is driven by rent-seeking.

Lemma 1

There exist $\eta^1, \eta^2, \eta^3 \in (0, 1)$ with $\eta^1 < \eta^3$ and $\eta^2 < \eta^3$ such that,
in equilibrium

- $\forall \eta \leq \eta^1$, if transfers could be directed exclusively towards recapitaliza-
tions (i.e., when transfers cannot be diverted towards rents), then the
supranational authority would partially fund additional recapitalizations
through an increase in the home country’s public debt;

- $\forall \eta \leq \eta^2$, the participation constraint for the politician binds given the
equilibrium policy $(\tau, x)$ set by the supranational authority to maximize
(4) subject to constraints (5) and (6).

- $\forall \eta \leq \eta^3$, the supranational authority partially funds additional recapital-
izations through an increase in the home country’s public debt;

Proof. In the Appendix. ■

Lemma 1 shows that the funding of interventions is directly linked to the
relative weights of countries at the supranational level. If the receiving coun-
try has a small weight compared to the donor country ($\eta \leq \eta^3$), then the
supranational authority prefers to pass on some of the costs of additional re-
capitalizations to this country, in order to decrease transfers and leave more
funds for public goods in the donor country. For $\eta \in (\eta^1, \eta^3)$, the receiving
country is forced to bear some of the cost of additional recapitalizations only
because of rent-seeking, as a means to make rents costlier for the politician.
If transfers could be directed only to recapitalizations, then any increase in
recapitalizations would be financed exclusively by transfers from the donor
country. Finally, if $\eta \leq \eta^2$, the politician’s participation constraint binds,
because the supranational authority cannot pass on to the home country as
much of the cost of interventions as it would prefer. The values of $\eta^1$, $\eta^2$ and $\eta^3$ depend on the other parameters of the model, but we can show that these regions are economically significant for a wide range of parameters.\(^9\)

The following Lemma allows us to emphasize the case when $\eta \in (\eta^1, \eta^2)$, the region where rent-seeking is costliest. In this case, the home country must bear the costs of intervention up to the point where the politician’s participation constraint binds, and all these costs are due exclusively to rent-seeking.

**Lemma 2** For $e^F$ sufficiently large, $\exists \underline{\varrho}, \overline{\varrho} \in \mathbb{R}$, with $\underline{\varrho} < \overline{\varrho}$, such that, if $|v''(r)|/|u^{H_H}(x, \theta)| \in [\underline{\varrho}, \overline{\varrho}]$, then $\eta^1 < \eta^2$, with $\eta^1, \eta^2$ defined in Lemma 1.

**Proof.** In the Appendix. ■

### 2.7 Household Welfare under a Partial Banking Union

The following result captures the main inefficiency of the model, which emerges when the home country carries a small enough weight in the decision problem of the supranational authority.

**Proposition 1** Suppose $\eta \leq \eta^2$, as defined in Lemma 1. A partial banking union increases donor household welfare, and it lowers household welfare in the home country.

The proof for this result is as follows. The supranational authority sets $\tau > 0$ and a binding intervention level $\underline{x}$. Then, the politician increases both rents and recapitalizations as a response to the binding limit $\underline{x}$. The increase in rents implies that $v(r^*) > v(r^0)$. Given the binding participation constraint, the politician gets the same utility under the partial banking union, with the higher rents, as he does without the partial banking union, with lower rents.

\(^9\)For instance, as shown in the Appendix, if public good provision is approximately the same in both countries, then $\eta^3 \approx 1/3$, while if public good provision in the donor country is half of that in the receiving country, and utilities take the logarithmic form, then $\eta^3 = 1/5$. The analysis for the case when transfers could be directed exclusively towards recapitalizations gives a similar result for $\eta^1$, given the expression for $\eta^1$ derived in the Appendix, and implicitly for $\eta^2$, given the conditions of Lemma 2 below.
The implication is that the supranational authority can constrain the politician to decrease the domestic public good compared to autarky. To see this, notice that the binding participation constraint for the politician is given by

\[(1 - \gamma)v(r^s) + \gamma U^H(x^s, g^{Hs}, g^{1Hs}) = (1 - \gamma)v(r^0) + \gamma U^H(x^0, g^{H0}, g^{1H0}).\]

Since \(v(r^s) > v(r^0)\) and \(u^H(x^s, \theta) > u^H(x^0, \theta)\), this implies \(U^H(x^s, g^{Hs}, g^{1Hs}) < U^H(x^0, g^{H0}, g^{1H0})\).

This result shows that the supranational authority is willing to accept some increase in rent-seeking in order to achieve an increase in donor household utility. This increase in rent-seeking comes at the cost of lower home household utility, as home households have to suffer the cost of higher rents. This decrease in utility cannot be avoided because the supranational authority does not have the right instruments to deter the politician from engaging in rent-seeking.

### 2.8 Domestic Institutions of Electoral Accountability

We now consider the effect of domestic electoral accountability on the terms of the partial banking union. With access to appropriate rewards and punishments, voters could develop a mechanism that limits the discretion of the politician and removes rent-seeking. However, electoral accountability mechanisms in the real world are limited in their ability to solve political economy distortions – they are mostly limited to removal from office. I consider a form of electoral accountability in this spirit, which builds on the models developed in Acemoglu (2005) and Besley (2007). Voters can decide politician removal at the end of the first period, after policies have been proposed, but before they are implemented and consumption takes place.\(^\text{10}\) If removed, the incumbent gets a minimum attainable utility \(V \to -\infty\) in the next period and is replaced with a politician chosen at random from a pool of possible politicians. The expected voter benefit from removing the politician is given by \(Z(b_1) > 0\), where \(Z(b_1)\) is concave, \(Z'(b_1) \geq 0\), and \(Z(b_1)\) is lower than the utility that a

\(^{10}\) Also, the model assumes that voters make the replacement decision collectively, and that they have solved any collective action problems ahead of the decision.
benevolent government would provide in the home country. $Z(b_1)$ represents a reduced-form representation of the expected household utility when a new politician is randomly selected from a pool of politicians with differing preferences over rents versus recapitalizations (different values of $\gamma$).\footnote{A micro-founded derivation of $Z(b_1)$ is provided in the Appendix.} We assume that

$$\frac{\partial Z(b_1)}{\partial b_1} < \frac{\partial u^H(x(b_1), \theta)}{\partial b_1} + \frac{\partial w(g^H)}{\partial b_1},$$

so that the marginal change in $Z(b_1)$ due to an increase in $b_1$ is smaller than the marginal effect of this increase on household utility. In the simplest case, $Z(b_1)$ is a constant. When $Z(b_1)$ is not constant, we allow the benefit from politician removal to depend on current economic conditions, summed up by debt $b_1$. If the politician takes on more debt, then the government budget is increased in the current period, which also increases voters’ expectation regarding what a new government could deliver in terms of goods and services. This electoral mechanism is represented by the following constraint:

$$u^H(x, \theta) + w(g^H) \geq Z(b_1). \quad (7)$$

Without the banking union, the politician in the home country chooses $\{r^0, x^0, g^{H0}, b_0^1\}$ to maximize (1) subject to (3a) with $\tau = 0$, (3c), (3d), and (7). The binding electoral constraint modifies the optimal choices of the politician compared to the baseline case. In response to voters’ electoral demands, the politician increases the provision of both recapitalization funds $x^0$ and public good $g^{H0}$. Then, given the first-order conditions to the politician’s problem and the budget constraint, the politician responds to the electoral demands by also increasing debt $b_0^1$ and decreasing rents $r^0$.

With a banking union, the politician faces the additional constraint (3b). A banking union leads to an increase in rents in the first period, due to the same forces as in the case without electoral accountability. If the politician’s participation constraint binds in equilibrium, the politician is indifferent to participating in the banking union, and so, the increase in rents under the banking union implies home household utility must decrease. The intuition is
that electoral accountability can guarantee more socially beneficial spending in the first period, but it cannot prevent the politician from borrowing more. Thus, the banking union still allows the politician to rent-seek more than under no banking union.

The other effect of electoral accountability is that it can lead to a decrease in the welfare of donor households.

**Proposition 2** Suppose \( \eta \leq \eta^2 \), as defined in Lemma 1, and there is domestic electoral accountability. If the politician places sufficiently high value on rents relative to the public good \( (w''(g^H)/v''(r)) \) is sufficiently small), then a partial banking union with electoral accountability lowers donor household welfare compared to a partial banking union without electoral accountability.

**Proof.** In the Appendix. ■

The result in Proposition 2 shows that, in a banking union, domestic institutions of electoral accountability can hurt household welfare in the donor country exactly in the situations in which they are most desirable for the home households: when rent-seeking is high. The intuition for this result is as follows. Electoral accountability has two opposing effects on the utility of donor households. First, the timing of elections allows the politician to increase household utility in the first period, but at the cost of higher public debt. The cost of additional public debt makes it more difficult for the supranational authority to incentivize the politician to increase recapitalizations. Second, the electoral constraint has a positive effect on donor households utility because of higher recapitalizations. Voters demand higher utility, and one way to satisfy their demands is through higher recapitalizations. Yet, if the politician has little propensity to spend more on recapitalizations rather than on rents and the public good, then the negative effect of electoral accountability on donor country utility dominates the positive effect: the benefit of electoral accountability through recapitalizations is small compared to the cost of higher debt for the donor country. This leads to lower donor household welfare compared to the case without electoral accountability.
2.9 Supranational Fiscal Rules

The second policy instrument that could be used to reduce rent-seeking is a limit on increases in public debt. Consider the baseline setup, without electoral accountability. As discussed above, any required increase in intervention leads the politician to increase public debt, in order to smooth the costs over both periods. Fiscal rules can help limit the degree to which increases in rent-seeking in the first period can be financed at the expense of less public good in the second period.

With binding fiscal rules, the politician chooses \( x^s, g^{Hs}, r^s \) given the budget constraint, with \( b^s_1 = b^{sFR}_1 \). This leads to the following first-order conditions and constraints:

\[
\begin{align}
(1 - \gamma) \nu'(r^s) &= \sigma R \gamma u^H(x^s, \theta) = \gamma w'(g^{Hs}), \\
r^s + x^s + g^{Hs} &
\leq e^H + \beta b^{sFR}_1 + \tau, \\
r^s + x^s &\geq \bar{x}, \\
g^{Hs}_1 &
\leq e^H - b^{sFR}_1.
\end{align}
\]  

(8a)-(8d)

The problem for the supranational authority is to choose transfer \( \tau \), intervention level \( \bar{x} \), and the the fiscal rule \( b^{sFR}_1 \) to maximize (4) subject to (8a)-(8d). The next result shows that the analysis from Proposition 1 carries through even in the presence of fiscal rules.

**Proposition 3** Suppose \( \eta \leq \eta^2 \), as defined in Lemma 1, and there is a fiscal rule that limits increases in public debt. A partial banking union lowers household welfare in the home country, and it increases household welfare in the donor country.

This result emerges because the second period is simply a consumption period, in which debt only affects the home country consumption. The role of debt is to balance public good provision between the first and second periods. Therefore, the objective of the supranational authority and that of the politician with respect to debt coincide. In the dynamic model presented in the next
section, this framework is enriched by having the supranational authority and the politician value debt differently, due to the additional effects of debt on the future utility of the donor households.

2.10 Fiscal Rules and Electoral Accountability

Consider now imposing electoral accountability and fiscal rules together. The supranational authority chooses transfer $\tau$, intervention $x$, and debt $b_1$ to maximize (4) subject to the home government’s policy choices and the participation constraint for each government. By controlling the increase in public debt, the supranational authority determines the budget available to the politician in the second period. The electoral constraint determines the utility the politician must guarantee for households in the first period. Then, rents in the first period become a residual given the constraints imposed by voters and the supranational authority. With rents constrained in this way, transfers from the donor country can be directed solely towards recapitalizations. This reduces the cost of recapitalizations compared with the case when transfers could be diverted towards rents. The result is summarized in the following proposition.

**Proposition 4** Suppose $\eta \leq \eta^2$, as defined in Lemma 1. With both electoral accountability and a fiscal rule that limits increases in public debt, a partial banking union achieves a Pareto improvement in household welfare compared to no banking union.

**Proof.** In the Appendix. ■

Proposition 4 shows how fiscal rules and electoral accountability can act as incentive complements. With these two instruments in place, rent-seeking is constrained, such that welfare does not decrease in the home country. In fact, for $\eta \in (\eta^1, \eta^2]$, household utility in the home country increases, as the supranational authority prefers to fund recapitalizations through transfers alone, without using increases in public debt. Fiscal rules cannot by themselves increase household welfare because they constrain overall government spending in the first period, and not rents in particular. Electoral accountability
alone cannot stop rent-seeking under the partial banking union because voters
cannot remove the rent-seeking incentives given to the politician through the
required intervention level; however, taken together, fiscal rules and electoral
accountability act as complements in reducing rents. They restrict the gov-
ernment budget without allowing for a decrease socially beneficial spending.

3 The Dynamic Model

In the two-period model, public debt had only a limited role, that of deter-
mining the domestic public good in the second period. This section develops
the model in a dynamic setting, in which future recapitalizations are possible.
Debt accumulation has different effects on the continuation utilities of home
and donor households, so the supranational authority’s preferences over debt
are different from those of the politician. The dynamic model also shows how
the costs and benefits of a banking union change in response to the evolution
of public debt.

We consider a setup similar to the one described in the two-period model.
Time is discrete, with periods $t = 0, \ldots, \infty$, and discount rate $\beta \in (0, 1)$ for all
agents. Each period, the households and the governments receive endowments
$\omega^j$ and $e^j$, $j = H, F, \ldots$ banks invest the deposits in risky projects, and these
projects are subject to liquidity shocks: an aggregate i.i.d. shock $\theta_t$ is realized,
$\theta_t \in \Theta = [\theta^0, \theta^N]$ with probability $f(\theta)$, and $\theta^0 > 0, \theta^N < 1$. Banks serve as a
vehicle for pooling together the household endowments and investing them in
projects, and their goal is to maximize expected household utility.

The donor government can decide each period whether to accept or reject
the supranational agreement offered that period, and this decision is denoted
by $\varrho_t^F \in \{0, 1\}$, with $\varrho_t^F = 1$ for acceptance. The donor government derives
utility

$$J_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ u^F(x_t, \theta_t) + w(e^F - \tau_t) \right].$$

A description of the household problem is provided in the Appendix. For the purposes
of simplicity, we assume that households decide to invest their entire endowment in deposits:
$\mathbb{E}_\theta \left[ u(\omega^j - i^j + R(1 - \theta)\bar{i}^j) \right] > u(\omega^j) \forall i^j \leq \omega^j.$

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The home government also decides participation in the agreement each period, denoted by $\rho \in \{0, 1\}$, and derives utility:

$$V_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma)v(r_t) + \gamma w(g^H_t) + \gamma u^H(x_t, \theta_t) \right].$$

The two-period model is extended such that electoral accountability is stochastic. This allows us to obtain a stronger result than in the two-period model, by showing that a Pareto improvement can be achieved even when there is no electoral accountability in some periods, but the frequency of electoral accountability is sufficiently high. Each period, with probability $\pi > 0$ an electoral shock is realized, denoted by $\chi = 1$, meaning that elections happen at the end of the period. With probability $1 - \pi$ the shock is $\chi = 0$, and the politician cannot be replaced at the end of the period.\(^{13}\) At the end of each period in which there is an election, voters can decide to replace the politician, a decision denoted by $\rho_t \in \{0, 1\}$, with $\rho_t = 1$ for replacement. If replaced, the incumbent receives the minimum attainable continuation utility, $V \rightarrow -\infty$. As in the two-period model, we denote by $Z(b, b')$ the demand for household utility in the current period, $u^H(x, \theta) + w(g^H)$, in case of an election. The function $Z(b, b')$ is concave in both its arguments, weakly increasing in $b'$ and weakly decreasing in $b$.\(^{14}\)

Lastly, the supranational authority offers $(\tau_t, x_t)$ each period, and has the expected utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \eta \left( u^H(x_t, \theta_t) + w(g^H_t) \right) + (1 - \eta) \left( u^F(x_t, \theta_t) + w(e^F - \tau_t) \right) \right].$$

\(^{13}\)The random electoral shock allows us to capture other ways through which voters express demands beyond regular elections, e.g., protests, recall elections etc.

\(^{14}\)As shown in the Appendix, $Z(b, b')$ can be micro-founded as the expectation of household utility in case of politician removal, when the pool of candidates is heterogeneous in their preferences for rents. This specification incorporates voters’ expectation of future policies, and it is consistent with forward-looking and fully rational voters.
3.1 Timing

At each date \( t; \) the timing of events is as follows:

1. The households receive their respective endowments \( \omega^H \) and \( \omega^F \), and the governments receive endowments \( e^H \) and \( e^F \); banks make investments in projects; shocks \( \theta_t \) and \( \chi_t \) are realized and observed by all agents;

2. The supranational authority offers an agreement \( (\tau_t, \xi_t) \) first to the donor country, then to the home country, and each government decides whether to accept or reject it;

3. The home government decides policies \( \{x_t, g^H_t, r_t, b_{t+1}\} \);

4. If \( \chi_t = 1 \), voters make politician replacement decision \( \rho_t \); if \( \rho_t = 1 \), the incumbent is replaced.

3.2 Equilibrium Concept

We consider the pure strategy Markov Perfect Equilibria of this game, in which strategies only depend on the current state of the world and not on the entire history of the game. The current state of the world in period \( t \) consists of the outstanding debt \( b_t \), the liquidity shock in the current period \( \theta_t \), and the electoral shock, \( \chi_t \). A Markov Perfect Equilibrium (MPE) is defined as a set of strategies \( \{\{\tau_t, \xi_t\}, \phi^F_t, \{\varphi_t, x_t, g^H_t, r_t, b_{t+1}\}, \rho_t\} \) such that these strategies depend only on the current payoff-relevant state of the economy \( \{b_t, \chi_t, \theta_t\} \) and on the prior actions within the same period, as described in the timing of events. Therefore, an MPE is given by a set of strategies \( \{\tau_t(b_t, \chi_t, \theta_t), \xi_t(b_t, \chi_t, \theta_t), \phi^F_t(b_t, \chi_t, \theta_t), \varphi_t(b_t, \chi_t, \theta_t), x_t(b_t, \chi_t, \theta_t), g^H_t(b_t, \chi_t, \theta_t), r_t(b_t, \chi_t, \theta_t), b_{t+1}(b_t, \chi_t, \theta_t), \rho_t(b_t, \chi_t, \theta_t)\} \), where for notational simplicity I do not explicitly introduce the dependence of each strategy on the actions already taken in the same period.\(^{15}\)

\(^{15}\)Formally, the set of strategies is written as \( \{\tau_t(b_t, \chi_t, \theta_t), \xi_t(b_t, \chi_t, \theta_t), \phi^F_t(\tau_t, \xi_t|b_t, \chi_t, \theta_t), \varphi_t(\tau_t, \xi_t|b_t, \chi_t, \theta_t), x_t(\tau_t, \xi_t, \phi^F_t|b_t, \chi_t, \theta_t), g^H_t(\tau_t, \xi_t, \phi^F_t|b_t, \chi_t, \theta_t), r_t(\tau_t, \xi_t, \phi^F_t|b_t, \chi_t, \theta_t), b_{t+1}(\tau_t, \xi_t, \phi^F_t|b_t, \chi_t, \theta_t), \rho_t(\tau_t, \xi_t, \phi^F_t, \varphi_t, x_t, g^H_t, r_t, b_{t+1}|b_t, \chi_t, \theta_t)\} \)
The focus on Markovian equilibria excludes any form of "consensual equilibria" in which the voters and the politician can use trigger strategies conditioned on the past realization of the investment shock $\theta$ or the electoral shock $\chi$. This restriction allows us to focus on the equilibria in which voters have limited means of punishing the incumbent, and electoral accountability is an imperfect tool for disciplining the incumbent.

The above framework with separable utility functions, discount factor $\beta < 1$ and bounded instantaneous utilities satisfies the conditions for the existence of a Markov Perfect Equilibrium to this game.

4 Analysis

4.1 Voters’ Problem

The problem is analyzed by studying each agent’s problem, in the reverse order of each period’s moves. Therefore, consider first the problem for the voters. If $\chi_t = 1$, the politician must offer voters at least $Z(b, b')$ in the current period in order to stay in power. If $\chi_t = 0$, the electoral constraint does not bind.

4.2 Home Government’s Problem

The politician decides domestic policy in the home country, given the partial banking union terms offered by the supranational authority. Each period, the state of the economy can be summarized by the outstanding debt $b_t$. The electoral shock $\chi_t$ and the shock $\theta_t$ are observed before policy is decided. Let $V(b, \chi, \theta, \tau, \bar{x})$ denote the maximum expected utility for the politician at the beginning of a period in which the state is given by $(b, \chi, \theta, \tau, \bar{x})$. The politician chooses a policy vector $\alpha = \{r, x, g^H, b'\}$ with $x \geq 0$, $g \geq 0$, $r \geq 0$, and a

---

$^{16}$As defined in Acemoglu (2005).
$^{17}$Given to the maximum attainable resources $e^H + \beta b$.
$^{18}$By Theorem 13.2 in Fudenberg & Tirole (1991).
decision to participate in the partial banking union $\varrho \in \{0, 1\}$ to solve:

$$
V(b, \chi, \theta, \tau, \underline{x}) = \max_{\alpha, \varrho} \{(1 - \gamma)v(r) + \gamma w(g^H) + \gamma u^H(x, \theta) \\
+ \beta \mathbb{E}_{\chi', \theta'}[V(b', \chi', \theta', \tau'(b', \chi', \theta'), \underline{x}'(b', \chi', \theta'))]\} 
$$

subject to

$$
\begin{align*}
\begin{aligned}
& r + x + g^H \leq e^H + \varrho \tau + \beta b' - b, \quad (10a) \\
& r + x \geq \varrho \underline{x}, \quad (10b) \\
& u^H(x, \theta) + w(g^H) \geq \chi Z(b, b'), \quad (10c) \\
& b' \in [\underline{b}, \bar{b}], \quad (10d) \\
& x \leq \theta (\omega^H + \omega^F). \quad (10e)
\end{aligned}
\end{align*}
$$

Constraint (10a) is the resource constraint of the economy. Constraint (10b) is the required intervention $\underline{x}$ as part of the partial banking union. Inequality (10c) is the minimum utility that must be provided to voters for the politician to stay in power. Finally, conditions (10d) and (10e) give the limits on debt and recapitalizations, respectively.

The politician’s problem can be reduced to the case where $\varrho = 1$ in all periods, given the equilibrium strategy of the supranational authority. The supranational authority is expected to follow the equilibrium policy functions in all future periods, while the current period’s agreement $(\tau, \underline{x})$ can be a deviation from that. If the politician does not participate in the agreement in the current period, let $\alpha^0 = \{x^0, r^0, g^{H0}, b^{00}\}$ denote the policies chosen by the politician in the current period. The outside option for the home government, $V^O(b, \chi, \theta)$, is derived by maximizing program (9) subject to (10a)-(10e), with $\tau = 0$ and $\underline{x} = 0$. The utility of the donor country in case of no agreement this period is given by

$$
J^O(b, \chi, \theta) = u^F(x^0, \theta) + w(e^F) \\
+ \beta \mathbb{E}_{\chi', \theta'}[J(b^0, \chi', \theta', \tau'(b^0, \chi', \theta'), \underline{x}'(b^0, \chi', \theta'))]. 
$$
### 4.3 Supranational Authority’s Problem

Lastly, the supranational authority seeks to maximize a weighted sum of household utilities. The supranational authority chooses to offer \((\tau, \bar{x})\), \(\bar{x} \geq 0, \, \tau \geq 0\), given the policies that will be chosen by the politician according to program (9) and the outside options described by \(V^O(b, \chi, \theta)\) and \(J^O(b, \chi, \theta)\). Denote the politician’s choices by \(\{g^H(b, \chi, \theta, \tau, \bar{x}), x(b, \chi, \theta, \tau, \bar{x}), r(b, \chi, \theta, \tau, \bar{x}), b'(b, \chi, \theta, \tau, \bar{x})\}\). Then the problem for the supranational authority is given by:

\[
S(b, \chi, \theta) = \max_{b, \chi, \theta} \left\{ \eta \left[ u^H(x, \theta) + w(g^H) \right] + (1 - \eta) \left[ u^F(x, \theta) + w(e^F - \tau) \right] \\
+ \beta \mathbb{E}_{\chi', \theta'} [S(b', \chi', \theta')] \right\}
\]

subject to

\[
V(b, \chi, \theta, \tau, \bar{x}) \geq V^O(b, \chi, \theta), \quad (13)
\]

\[
J(b, \chi, \theta, \tau, \bar{x}) \geq J^O(b, \chi, \theta). \quad (14)
\]

Constraint (13) represents the participation condition for the politician, and constraint (14) is the participation constraint for the donor government, given the outside option described in (11).

### 4.4 Optimal Domestic Policy Choices

In order to characterize the politician’s problem, the analysis restricts attention to the cases in which the value functions for the politician and the supranational authority are concave. The existence of functions \(v(\cdot), u(\cdot),\) and \(w(\cdot)\) that satisfy the conditions necessary for the value functions to be concave and differentiable is established in the following Lemma.

**Lemma 3** There exist concave functions \(v(\cdot), u(\cdot),\) and \(w(\cdot)\) such that the politician’s value function \(V(b, \chi, \theta, \tau(b, \chi, \theta), \bar{x}(b, \chi, \theta))\) is concave and differentiable for \(b \in (b, \bar{b})\) given the equilibrium policy functions \(\tau(b, \chi, \theta)\) and \(\bar{x}(b, \chi, \theta)\), and the supranational authority’s value function \(S(b, \chi, \theta)\) is concave and differentiable for \(b \in (b, \bar{b})\) given the equilibrium policies chosen by the politician.
Proof. In the Appendix. □

Lemma 3 allows for a characterization of the politician’s problem. Denote by $\lambda(\chi, \theta)$, $\kappa(\chi, \theta)$, $\nu(\chi, \theta)$ and $\varphi(\chi, \theta)$ the Lagrange multipliers on constraints (10a), (10b), (10c), and (10e), respectively. The first-order conditions for an internal solution with respect to $r$, $g^H$, and $b'$ are:

$$
\lambda(\chi, \theta) - \kappa(\chi, \theta) = (1 - \gamma)v'(r),
$$

(15)

$$
\lambda(\chi, \theta) - \kappa(\chi, \theta) + \varphi(\chi, \theta) = \gamma (1 + \nu(\chi, \theta)) \frac{\partial u^H(x, \theta)}{\partial x},
$$

(16)

$$
\lambda(\chi, \theta) = \gamma (1 + \nu(\chi, \theta)) w'(g),
$$

(17)

$$
\lambda(\chi, \theta) - \nu(\chi, \theta) \left[ \gamma \frac{\partial Z(b, b')}{\partial b'} \right] = \mathbb{E} \left[ - \frac{\partial V(b', \chi', \theta', \tau', \xi')}{\partial b'} \right].
$$

(18)

The above conditions show that the effects described in the two-period model translate to the dynamic environment. The effect of the electoral constraint on public debt comes through the term $-\nu(\chi, \theta) \left[ \gamma \frac{\partial Z(b, b')}{\partial b'} \right]$, which captures the fact that taking on more public debt increases the electoral demands, and the term $\lambda(\chi, \theta)$, which captures the fact that higher debt relaxes the budget constraint in the current period. We make the following assumption about the relative effect of debt on the function $Z(b, b')$.

Assumption 3 The following condition holds at the equilibrium solution:

$$
\lambda(1, \theta) - \nu(1, \theta) \left[ \gamma \frac{\partial Z(b, b')}{\partial b'} \right] > \lambda(0, \theta).
$$

Assumption 3 gives the condition under which public debt increases in equilibrium when the electoral constraint binds. In the Appendix, we can show that this condition holds when $Z(b, b')$ is the expected household utility after politician removal, when the pool of candidates consists of a fraction $q \in (0, 1)$ of rent-seeking politicians and a fraction $(1 - q)$ of benevolent politicians.

\[19\]While the conditions of the Lemma restrict the set of possible utility functions, this approach helps provide a tractable framework under which the problem can be analyzed. In Section 5.3.1, the problem is illustrated numerically under a logarithmic form for $v(\cdot)$, $u(\cdot)$, and $w(\cdot)$. 

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Next, we show that there exist values of \( \eta \) and \( \bar{b} \) at which the participation constraint for the home government always binds in equilibrium.

**Assumption 4** Given \( \{\eta, e^H\} \in [0, 1] \times [0, \infty) \), the participation constraint for the politician binds \( \forall \theta \in \Theta, \chi \in \{0, 1\} \) and \( (\tau, z) \) set by the supranational authority to maximize (12) subject to constraints (13) and (14).

Assumption 4 implies a sufficiently small weight \( \eta \) on the home country, along with a sufficiently small upper limit on debt, since \( \bar{b} \leq \frac{e^H}{1 - \beta} \). As discussed in the two-period model, the set of values of \( \eta \) for which the participation constraint of the politician binds can be shown to be economically significant for a wide range of utility functions.

### 4.5 Partial Banking Unions and Household Welfare

We begin the analysis of the dynamic model by considering the change in household welfare under a partial banking union compared to no banking union. Higher recapitalizations are beneficial for the consumers of both countries; however, a partial banking union might also lead to increased rent-seeking, which could make home country consumers worse off. The intuition for why welfare might decrease is similar to the one presented in the two-period model: the supranational authority does not value the home country public good as much as the home country consumers, so it is willing to accept a larger decrease in the public good in exchange for higher recapitalizations. Still, the dynamic model introduces another element in the decision problem of the supranational authority. Now the supranational authority places a different weight on decreases in the home public good in the current period versus decreases in the public good in future periods. This happens because a decrease in the home country public good today only affects the utility of home country consumers, while a decrease in future public good provision also implies a decrease in future recapitalizations, through the effect of higher debt. Therefore, Proposition 1 extends to the dynamic environment, supported by weaker forces.
Proposition 5 A partial banking union lowers expected household welfare in the home country, and it increases household welfare in the donor country, compared to no banking union.

Proof. In the Appendix.

Home household welfare decreases because the politician is kept indifferent to participating in the banking union, and the politician’s incentives differ from those of households. The politician gains from increasing rent-seeking under the banking union, at the cost of public good provision. This happens because the incentives for increasing recapitalizations can only be given through transfers and the required intervention level, and both of these policies also act towards making rent-seeking more attractive for the politician.

The dynamic version of the model shows the effect of public debt on household welfare over time. The future utility of donor households is a function of the home country’s public debt. When outstanding debt is higher, the cost of taking on more debt is borne by households in both countries, because their expected future consumption decreases. The higher cost of debt induces the supranational authority to prefer using transfers over further increases in public debt. This shifts the burden of recapitalizations towards the donor country.

5 Electoral Accountability and Fiscal Rules

5.1 Higher Electoral Accountability

This section considers the effect of higher electoral accountability on household welfare. The strength of electoral accountability in this model is given by the size of voters’ demands $Z(b,b')$. In terms of the micro-foundations for $Z(b,b')$, this can be interpreted as a higher fraction of non-rent-seeking candidates in the pool of potential politician replacements. The next Proposition gives the dynamic extension to Proposition 2.

Proposition 6 If the politician’s preference for rent-seeking relative to the public good is sufficiently strong ($v''(r)/w''(g^H)$ is sufficiently large), then
higher electoral accountability (an increase in $Z(b,b')$) in the home country lowers donor household welfare under the partial banking union.

Proof. In the Appendix. ■

The intuition for the change in expected welfare is as follows. The need to provide households with a higher expected utility leads the incumbent to increase public debt, since more funds are necessary to cover the voter demands in the current period. Higher debt also increases the marginal cost of any additional recapitalizations that would be required under the banking union. Therefore, donor households face a higher cost of recapitalizations, through higher transfers. As in the two-period model, the condition that rent-seeking is sufficiently high ensures that the positive effect of electoral accountability – more recapitalizations – does not outweigh the negative effect of higher debt.

The above results show that an improvement in domestic electoral institutions is not by itself sufficient to increase welfare in both countries. While desirable from the perspective of home household welfare, it can have a negative effect on the donor country welfare because it makes it harder to obtain cost-sharing in a partial banking union.

5.2 Partial Banking Unions and Fiscal Rules

The results obtained above show that increases in public debt can lead to decreased household welfare. A natural question is then whether fiscal rules that constrain public debt could increase welfare. The type of fiscal rules considered are limits on the increase in public debt, chosen by the supranational authority. For binding fiscal rules, this is equivalent to assuming that the supranational authority controls debt. As before, the decision over the composition of domestic spending ($x$, $g^H$, and $r$) belongs to the home government. Let the choices of the politician be denoted by \{ $x(b,b',\chi,\theta,\tau,\underline{x})$, $r(b,b',\chi,\theta,\tau,\underline{x})$, $g^H(b,b',\chi,\theta,\tau,\underline{x})$ \}. The problem for the
supranational authority is given by:

\[
S(b, \chi, \theta) = \max_{\xi, \tau, \theta'} \{ \eta [u^H(x, \theta) + w(g^H)] + (1 - \eta) [u^F(x, \theta) + w(e^F - \tau)] \\
+ \beta \mathbb{E} [S(b', \chi', \theta')] \},
\]

subject to (13), (14) and (10d).

**Lemma 4** The supranational authority’s value function \( S(b, \chi, \theta) \) is concave and differentiable in \( b \in (b, \bar{b}) \).

**Proof.** In the Appendix.  

When there are no elections, the politician retains some discretion in choosing current period policies. The supranational authority controls debt increases, but it cannot offer incentives for increasing recapitalizations without these incentives also acting towards increasing rents. When there are elections (\( \chi = 1 \)), the voters and the supranational authority together drastically limit politician discretion. Voters constrain the composition of intra-period spending, while the supranational authority controls how much the politician is allowed to borrow. Together, the two instruments can be used to offer incentives for higher recapitalizations without enabling higher rent-seeking.

**Proposition 7** In a partial banking union with fiscal rules, in periods with electoral accountability, rent-seeking is (weakly) lower than under no partial banking union.

**Proof.** In the Appendix.  

The intuition for the above result is based on the supranational authority’s and the voters’ ability to constrain the politician’s choices when fiscal rules and electoral accountability are in place. In these periods, fiscal rules can be used to constrain the available government budget, while the electoral constraint ensures that the spending that benefits households is not decreased. In order to separate the case when rents strictly decrease under the partial banking union, we derive the dynamic equivalent of Lemma 1. Denote by \( \eta^{2d} \) the largest value of \( \eta \) feasible for a given \( e^H \) such that Assumption 4 is satisfied.
Lemma 5  For each $b \in [\underline{b}, \overline{b}]$, there exist $\eta^{1d}(b)$ and $\eta^{3d}(b) \in (0, 1)$ with $\eta^{1d} < \eta^{3d}$ such that, in equilibrium

- $\forall \eta \leq \eta^{1d}$, if transfers in the current period could be directed exclusively towards recapitalizations, then the supranational authority would partially fund recapitalizations through higher public debt;

- $\forall \eta \leq \eta^{3d}$, the supranational authority partially funds recapitalizations in the current period through higher home country public debt;

Proof. In the Appendix.

Lemma 5 shows that, if the home country carries a low weight for the supranational authority ($\eta < \eta^{3d}$), then the supranational authority has an incentive to increase debt in the home country. Yet, if rent-seeking can be constrained, the supranational authority prefers a decrease in public debt for a wider range on weights $\eta$ ($\eta > \eta^{1d}$).

Proposition 8  If $\eta^{1d} < \eta^{2d}$ and $\eta \in (\eta^{1d}, \eta^{2d}]$, then there exists threshold $\pi^*$ for the frequency of periods of electoral accountability, such that a partial banking union achieves a Pareto improvement in household welfare $\forall \pi > \pi^*$.

Proof. In the Appendix.

Proposition 8 shows that a partial banking union leads to an increase in expected household welfare, if the periods of electoral accountability are sufficiently frequent. During these periods, fiscal rules can be used to decrease rents and public debt, leading to higher home household utility. Therefore, a Pareto improvement can be obtained as long as there is sufficient electoral accountability, even if the politician is not constrained by voters in every period. The condition that $\eta^{1d} < \eta^{2d}$ implies that the increase in public debt under the banking union is due exclusively to rent-seeking and not due to the supranational authority placing too little weight on the home country.

The above results shed light on the interaction between domestic electoral institutions and supranational agreements. First, domestic electoral accountability is needed in order to achieve a Pareto improvement. Second, while the
supranational authority needs electoral accountability in order to lower rent-seeking, higher voter demands in the home country decrease donor household welfare, as shown in Proposition 6.

5.3 Debt Dynamics

In this section, we examine the evolution of public debt. The terms of the banking union affect the level of debt, and this in turn affects the future terms of the partial banking union and the path of public debt.

**Proposition 9**  The equilibrium distribution of public debt converges to a unique nondegenerate invariant distribution over $[h, \bar{b}]$.

**Proof.** In the Appendix.

The politician uses debt to smooth the costs of interventions and public good provision over time. When a high liquidity shock $\theta$ is realized or when $\chi = 1$, current spending increases, and the politician takes on more debt in order to finance higher current spending. When the liquidity shock $\theta$ is low, or the electoral constraint is not binding ($\chi = 0$), the need for government spending in the current period is lower, so the politician takes on less debt.

Given the binding participation constraint, the politician takes on part of the cost of additional recapitalizations through decreases in the public good and higher debt. As debt increases, the share of the cost of recapitalizations borne by the politician decreases, since the costs of additional recapitalizations are higher for the more constrained politician. Then, the donor country must cover a higher relative share of the cost of recapitalizations, and so, the benefits from a banking union decrease for the donor country. This model highlights why partial banking unions may be harder to implement in high debt environments, if much of the bargaining power rests with the donor country.

5.3.1 Numerical illustration

The evolution of the public debt over time and the effects of fiscal rules are illustrated using a numerical simulation. Consider the following specifications
for the utility functions: \( v(r) = r^d \log(r) \), \( w(g) = g^d \log(g + g^c) \), \( u^j(x, \theta) = x^d \log(R(1 - \theta) \omega_j + Rx^j) \), \( j = H, F \), and \( r^d, x^d, g^d, g^c \in \mathbb{R}_+ \). The parameters are taken as \((r^d; x^d; g^d; g^c; R; \sigma; \gamma; \beta; b; \bar{b}) = (0.02; 0.05; 0.46; 10; 1.02; 0.75; 0.95; 0.7; 0; 1)\) and \( \eta = 0 \). The endowments are given by \((\omega^H; \omega^F; \epsilon^H; \epsilon^F) = (1; 1; 1; 5)\) The weight \( \eta = 0 \) reflects the limit case in which the supranational authority is simply a proxy for the donor country. We assume that \( \theta \) can only take two values, \( \theta^H = 0.2 \) and \( \theta^L = 0.1 \), with probabilities \( f^H = 0.1 \) and \( f^L = 0.9 \). The probability of \( \chi = 1 \) is \( \pi = 0.1 \).

![Public Debt](image)

**Figure 1:** Fiscal rules and the evolution of public debt

First, we compare the path of debt under no banking union to that under a partial banking union, both with and without fiscal rules. Figure 1 shows the path of debt under a sequence of realized \((\theta, \chi)\) shocks over 50 periods, starting from 0 initial debt. The vertical axis measures the public debt each period relative to the home country’s endowment. As shown in Figure 1, high liquidity shocks or high electoral shocks lead to temporary increases in debt, represented by the spikes seen on the path of debt. The combination of high liquidity and high electoral shocks leads to a much larger increase in debt, as
shown, for example, in period 13. The role of fiscal rules in limiting the increase in public debt becomes apparent. Public debt still increases in response to high liquidity or electoral shocks, but the overall debt accumulation is smaller, as the supranational authority can use fiscal rules to limit rents.

Figure 2 illustrates the complementarity of fiscal rules and electoral accountability: rents decrease even more in the periods of electoral accountability when fiscal rules are in place. When the electoral constraint is not binding, rents are the same with or without fiscal rules, which shows that fiscal rules are effective only together with electoral accountability.

6 Conclusion

This paper presented a model of a partial banking union with domestic rent-seeking and showed that implementing such a banking union can reduce household welfare. Higher electoral accountability meant to reduce rent-seeking may do so at the cost of lower donor country welfare. Strengthening the bank-
ing union with fiscal rules can limit rent-seeking, but such limits also reduce the ability of governments to engage in desirable public spending, which can decrease welfare in the receiving country. These results suggest that policies aimed at tackling one source of inefficiency might backfire by augmenting the other incentive problems. Yet, the model also shows that jointly set intervention rules and fiscal rules, optimized for each other, could achieve a Pareto improvement in consumer welfare. These implications seem relevant for the proposed banking union in the Eurozone, in which not all decisions regarding interventions in the banking sector can be centralized.

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A Appendix (For Online Publication)

A.1 Household Investment and Banks

This section endogenizes the household investment decisions, describes the
banks in more detail and presents the necessary assumptions made such that
households decide to invest their entire endowment in banks.

A.1.1 Households

Every household receives endowment $\omega^j, j = H, F$, at the beginning of
each period. It can use this endowment for direct consumption, or it can
decide to make a risky deposit $i_t^j$ in a bank. Households do not have access to
any storage technology. The deposit has a risky return that depends the an
aggregate shock $\theta_t$. The return is in terms of consumption goods, denoted as
$c(i_t^j, \theta_t)$. Their instantaneous utility is given by $u(\omega^j - i_t^j + c(i_t^j, \theta_t) + w(g_t^j)$.

A.1.2 Banks

Banks hold identical, risky investment projects. They do not have any
equity and can fund projects exclusively using household deposits. Moreover,
I make the simplifying assumption that the banks are owned by the same
households that hold deposits in them, so that the objective of the bank is to
maximize the expected household utility from private consumption. The in-
vestment technology exhibits constant returns to scale. The initial investment
$(i_t^H + i_t^F) \geq 0$ determines the size of the project. The project return is subject
to uncertainty. Following investment, an aggregated i.i.d. shock $\theta_t \in \Theta$ is
realized. After the shock, a fraction $\theta_t$ of the investment is lost, while the
remaining $(1 - \theta_t)$ fraction of the project is intact. The intact portion of the
project has a rate of return $R > 1$ in the next period. The distressed portion
does not produce any returns, unless additional funds are reinvested. Following
the observation of $\theta_t$ and prior to the investment project completion, the
bank can reinvest $x_t$ new funds into the project, such that the total size of
the project is at most equal to the initial size: $x_t \leq \theta_t (i_t^H + i_t^F)$. Since the
households and banks do not have access to any storage technology and there is no loan market for the bank to access new reinvestment funds, all reinvestment funds \(x_t\) must be provided by the government. The government is the only agent who has access to loan markets and also has an endowment that cannot be initially invested in private projects. Therefore, the government is the only provider of liquidity in case of a shock to the project. The project then returns \(R((1 - \theta_t)(i_t^H + i_F^H) + x_t)\) consumption units. This timing assumption precludes the banks from having access to next period’s household endowment. In terms of household consumption, the investment \(i_t\) made by each households returns \(R((1 - \theta_t)i_t^H + x_t)\) consumption units, depending on the reinvestment \(x_t\) made by the government.

A.1.3 The Household Problem

Each household is choosing whether to invest some part of its endowment. Given an investment \(i\), households receive expected utility: \(\mathbb{E}_\theta[u(\omega^j - i^j + R(1 - \theta)i^j + Rx^j)]\), where \(j = H, F\). To simplify the problem, the following assumption is made so that households always prefer to fully invest their endowment rather than directly consume. This requires assuming that the rate of return \(R\) is high enough such that the following condition holds: \(\mathbb{E}_\theta[u(\omega^j - i^j + R(1 - \theta)i^j)] > u(\omega^j), \forall i^j \leq \omega^j\).

A.1.4 The Electoral Process

If an election takes place at the end of the period, and voters decide politician replacement, a new politician is randomly selected from a pool of possible politicians. With probability \(q > 0\), the new politician will be identical to the previous one. With probability \(1 - q\) the new politician’s objective will be to maximize household utility. A key limiting factor for voters is that, due to the timing of elections, the replacement decision is made after debt has been decided by the incumbent, and this level of debt cannot be changed by the newly elected politician. Yet, since elections take place before consumption, the current period policies can still be modified by a newly elected politician.
Moreover, we assume that the terms of a partial banking union (the transfer and the intervention level) are no longer binding if a new politician is elected. These assumptions rely on the observation that government budgets (including the level of indebtedness) are generally set in advance, while current public expenses can be easily modified (by re-allocating funds from one agency to another). Moreover, it is not uncommon that, once a government falls, the terms of international transfers are re-negotiated.

In the two-period model, the electoral mechanism described above is then represented by the following electoral constraint, which reflects the problem faced by voters:

\[ u^H(x, \theta) + w(g^H) \geq q \left( u^H(x^p, \theta) + w(g^{Hp}) \right) + (1 - q) \left( u^H(x^*, \theta) + w(g^{H*}) \right), \]  

(20)

where \((x^p, g^{Hp})\) are the policies preferred by a rent-seeking politician, and \((x^*, g^{H*})\) are the policies preferred by a households given budget \((e^H + \beta b_1)\). Then,

\[ Z(b_1) \equiv q \left( u^H(x^p, \theta) + w(g^{Hp}) \right) + (1 - q) \left( u^H(x^*, \theta) + w(g^{H*}) \right), \]

and it immediately follows that \(Z(b_1)\) is an increasing, concave function of \(b_1\).

In the dynamic model, denote by \(\{r^*_t(b_t, \theta_t), x^*_t(b_t, \theta_t), g^{H*}_t(b_t, \theta_t), r^*_{t+1}(b_{t+1}, \theta_{t+1})\}_{t=0}^\infty\) the policies chosen when a benevolent politician maximizes home household utility each period and a supranational authority sets the terms of the partial banking union according to problem (4). The expected voter utility in case of replacement is given by

\[ q \left( u^H(x^p, \theta) + w(g^{Hp}) + \beta E_{\lambda', \theta'} \left[ U^H(b') \right] \right) + (1 - q) \left( u^H(x^*, \theta) + w(g^{H*}) + \beta E_{\theta'} \left[ U^{H*}(b') \right] \right). \]  

(21)

where \((x^p, g^{Hp})\) are the policies preferred by a rent-seeking politician in the current period, \(U^H(b')\) denotes the continuation utility for households given debt \(b'\) and a rent-seeking politician, \((x^*, g^{H*})\) are the policies preferred by households in the current period, and \(U^{H*}(b')\) is the expected continuation
utility for households given debt $b'$ and a benevolent politician. Then, the electoral constraint faced by the politician is:

$$u^H(x, \theta) + w(g^H) + \beta \mathbb{E}_{\chi', \theta'} [U^H(b')] \geq q(u^H(x^p, \theta) + w(g^{Hp}) + \beta \mathbb{E}_{\chi', \theta'} [U^H(b')]$$

Define

$$Z(b, b') \equiv q(u^H(x^p, \theta) + w(g^{Hp})) - (1 - q) \beta \mathbb{E}_{\chi', \theta'} [U^H(b')]$$

which will be used to denote the constraint on household utility in the current period, $u^H(x, \theta) + w(g^H)$, as a function of public debt.

Next, we show that the condition in Assumption 3 holds $\forall q \in (0, 1)$. Denote by $\{x, g, r, b'\}$ the politician’s policy choices in the absence of an electoral constraint. By the first-order conditions (15)-(18), the choice of debt $b'$ is higher for a rent-seeking politician than for a benevolent politician, for whom $\gamma = 1$. Therefore, given the concavity of the utility functions,

$$(1 - q) \frac{\partial [u^H(x^*, \theta) + w(g^{H*}) + \beta \mathbb{E}_{\theta'} [U^{H*}(b')]}{\partial b'} < 0,$$

where $u^H(x^*, \theta), w(g^{H*})$, and $\mathbb{E}_{\theta'} [U^{H*}(b')]$ denote the policies implemented by a benevolent government given debt $b'$.

Also, notice that, for the other part of the electoral constraint,

$$\frac{\partial [qu^{Hp}(x, \theta) + qw(g^{Hp}) - (1 - q) \beta \mathbb{E}_{\theta'} [U^{Hp}(b')]}{\partial b'} > 0,$$

where $u^H(x^p, \theta), w(g^{Hp})$, and $\mathbb{E}_{\theta'} [U^{Hp}(b')]$ denote the policies implemented by a rent-seeking politician given debt $b'$. 

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Then, at $b'$, $u^H(x, \theta) = u^H(x^p, \theta)$, $w(g^H) = w(g^H_p)$,

$$\mathbb{E}_{\theta'} [U^H(b')] = \mathbb{E}_{\theta'} [U^{Hp}(b')] ,$$

and

$$\frac{\partial}{\partial b'} \left[ u^H(x, \theta) + w(g^H) + \beta\mathbb{E} [U^H(b')] \right] >$$

$$p \frac{\partial}{\partial b'} \left[ u^H(x^p, \theta) + w(g^{Hp}) + \beta\mathbb{E} [U^{Hp}(b')] \right]$$

$$+ (1 - q) \frac{\partial}{\partial b'} \left[ u^H(x^*, \theta) + w(g^{Hp*}) + \beta\mathbb{E} [U^{Hp*}(b')] \right].$$

Therefore, the increase in household utility due to an increase in debt is higher than the increase in the electoral constraint. Then, debt increases whenever the electoral constraint is not satisfied at $\{x, g, r, b'\}$, i.e., whenever $q < 1$.

**A.2 Proofs**

**A.2.1 Proof of Lemma 1**

**Part 1**

Denote by $\{x^R, \tau^R\}$ the solution to problem (4) given the additional restriction that $\tau \geq \tau - (x^0 + r^0)$, i.e., the increase in interventions is fully financed by transfers. As above, $\{x^0, r^0, g^{H0}, g^{H1}\}$ denote the politician’s choices in autarky.

From the first-order conditions to the politician’s problem,

$$w'(g^H) = w'(g^{H1}).$$

A decrease of $\varepsilon > 0$ in both $g^H$ and $g^{H1}$, keeping all other policy choices fixed, would allow for an increase in $x$ of $(1 + \beta)\varepsilon$. Also, an increase of $\varepsilon$ in $\tau^R$ would allow for an increase in $x$ of $\varepsilon$. It then follows that the supranational authority prefers to partially fund recapitalizations through an increase in
home country public debt if

\[ \eta w'(g^{H0}) < (1 - \eta)w'(e^F - \tau^R). \]  

(22)

Given a binding constraint \( \tau^R = x^R - (x^0 + r^0) \), the problem for the supranational authority yields the first-order condition for \( \tau^R \):

\[ \frac{(1 - \eta)}{\eta}w'(e^F - \tau^R) = \left[ u^H(x, \theta) + \frac{(1 - \eta)}{\eta}w^F(x, \theta) \right] \frac{\partial x}{\partial \tau}. \]  

(23)

Therefore, \( \tau^R \) is an increasing function of \( \eta \). Also, the above implies that \( \frac{(1 - \eta)}{\eta}w'(e^F - \tau^R) \) is monotonically decreasing in \( \eta \). Moreover, condition (22) is satisfied at \( \eta = 0 \) and \( \eta w'(g^{H0}) > (1 - \eta)w'(e^F - \tau^R) \) at \( \eta = 1 \), which, given the continuity and monotonicity of \( \frac{(1 - \eta)}{\eta}w'(e^F - \tau^R) \) implies that \( \exists \eta^3 \in (0, 1) \) such that condition (22) is satisfied for \( \eta \leq \eta^3 \). Then,

\[ \eta^3 = \frac{1}{1 + w^*(g^{H0})/w^*(e^F - \tau^R)}. \]  

(24)

**Part 2**

Assume now that the supranational authority could direct all transfers towards recapitalizations, so that the politician cannot extract additional rents. This is equivalent to considering the solution to problem (4) given the additional restrictions that \( \tau \geq x - (x^0 + r^0) \) and \( r \leq r^0 \). Denote this solution by \( \{x^{Rr}, \tau^{Rr}\} \). In this case, by Assumption 2, full recapitalizations are achieved, so

\[ x^{Rr} = \theta(\omega^H + \omega^F) + r^0. \]

It follows that the supranational authority prefers to fund all recapitalizations through transfers if

\[ \eta w'(g^{H0}) > (1 - \eta)w'(e^F - \theta(\omega^H + \omega^F) + x^0), \]  

(25)

since full recapitalizations require \( \tau^{Rr} = \theta(\omega^H + \omega^F) - x^0 \).
Denote by $\eta^1$ the value of $\eta$ at which condition (25) holds with equality:

$$\eta^1 = \frac{1}{\frac{1}{w'(g^{H0})} + 1}. \quad (26)$$

It follows immediately that $\eta^1 \in (0, 1)$ and condition (25) is satisfied $\forall \eta \geq \eta^1$.

For $\eta < \eta^1$,

$$\frac{(1 - \eta)}{\eta} w'(e^F - \theta(\omega^H + \omega^F) + x^0) < \left[ u^H'(\theta(\omega^H + \omega^F), \theta) + \frac{(1 - \eta)}{\eta} u^{F'}(\theta(\omega^H + \omega^F), \theta) \right]$$

$$\left[ u^H(x, \theta) + \frac{(1 - \eta)}{\eta} u^{F'}(x, \theta) \right],$$

$\forall x < \theta(\omega^H + \omega^F)$. 

For $x < \theta(\omega^H + \omega^F)$, if $\frac{\partial x}{\partial \tau}$ is sufficiently large such that

$$\left[ u^H(x, \theta) + \frac{(1 - \eta)}{\eta} u^{F'}(x, \theta) \right] \frac{\partial x}{\partial \tau} > \frac{(1 - \eta)}{\eta} w'(e^F - \theta(\omega^H + \omega^F) + x^0), \quad (27)$$

then (23) implies $\tau^R < \tau^R$, and

$$w'(g^{H0}) = \frac{(1 - \eta^2)}{\eta^3} w'(e^F - \tau^R) = \frac{(1 - \eta^1)}{\eta^2} w'(e^F - \theta(\omega^H + \omega^F) + x^0).$$

so $\eta^1 < \eta^3$.

**Part 3**

Consider now the original problem for the supranational authority. For any $\tau \leq e^F$, denote by $x^\tau$ the value of $x$ at which the participation constraint for the politician, constraint (6), holds with equality. If there is no full recapitalization ($x < x^{MAX}$)\footnote{If there is full recapitalization in equilibrium ($x = x^{MAX}$), the participation constraint for the politician binds if the marginal cost of transfer $\tau$ is higher than the marginal cost of debt $b_1: \beta \eta [w'(e^H + \tau + \beta b_1 - x^\tau) - w'(e^H - b_1)] < (1 - \eta) w'(e^F - \tau)$. At $\eta = 0$, the condition becomes $0 < w'(e^F - \tau)$, which holds for all positive transfers $\tau$. At $\eta = 1$, the condition is $\beta [w'(e^H + \tau + \beta b_1 - x^\tau) - w'(e^H - b_1)] < 0$. The above inequality requires}, then, the participation constraint for the politician
binds in equilibrium at transfer $\tau$ when the marginal benefit for the supranational authority from increasing $x$ is larger than the benefit from decreasing $\tau$. In that case, the supranational authority could achieve higher utility by marginally decreasing $\tau$ and increasing $x$. The condition for the politician’s participation constraint to bind is:

$$\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \frac{\partial x^*}{\partial x} - \eta w' \left( e^H + \frac{\tau - x^*}{1 + \beta} \right) \geq$$

$$\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \frac{\partial x}{\partial \tau} + \eta w' \left( e^H + \frac{\tau - x^*}{1 + \beta} \right) - (1 - \eta) w' (e^F - \tau).$$

The above condition can be re-written as:

$$\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \left( \frac{\partial x}{\partial x^*} \right) \left( \frac{\partial x}{\partial \tau} \right) - 2\eta w' \left( e^H + \frac{\tau - x^*}{1 + \beta} \right) + (1 - \eta) w' (e^F - \tau) \geq 0. \quad (28)$$

At $\eta = 0$, condition (28) holds since

$$\left( \frac{\partial x}{\partial x^*} - \frac{\partial x}{\partial \tau} \right) \geq 0$$

given constraints (3a) and (3b) in the politician’s problem.

At $\eta = 1$, condition (28) does not hold, because, from the first-order conditions to the politician’s problem, it follows that $\frac{\partial u^H(x, \theta)}{\partial x} \leq w' \left( e^H + \frac{\tau - x^*}{1 + \beta} \right)$ and $\left( \frac{\partial x}{\partial x^*} - \frac{\partial x}{\partial \tau} \right) \leq 1 < 2$ (since $\frac{\partial x}{\partial x} \leq 1$ and $\frac{\partial x}{\partial \tau} \geq 0$).

Finally, the left-hand side of condition (28) is continuous in $\eta$. It it also

$$\tau + (1 + \beta) b_1 - x^* > 0,$$

which is equivalent to $g^H > g^H_1$. If this holds, then the politician’s participation constraint binds $\forall \eta$. Otherwise, $\eta^* = \frac{1}{1 + \beta} \left[ \frac{w'(e^H + \tau + b_1 - x^*)}{w'(e^F - \tau)} - \frac{\tau - x^*}{w'(e^H - b_1)} \right]$.
monotonically decreasing in \( \eta \) since

\[
\frac{\partial u^H(x, \theta)}{\partial x} \left( \frac{\partial x}{\partial x} - \frac{\partial x}{\partial \tau} \right) \leq \frac{\partial u^H(x, \theta)}{\partial x} \leq w^r \left( e^H + \frac{\tau - x^r}{1 + \beta} \right),
\]

\( \forall \tau \in [0, e^F] \). Therefore, \( \exists \eta^2 \in (0, 1) \) such that condition (28) holds \( \forall \eta \leq \eta^2 \).

We can derive \( \eta^2 \) as the value at which the supranational authority is indifferent to using home country public debt or to further increase \( \tau \):

\[
\eta^2 = \frac{1}{1 + \frac{(1+\beta)w^r(g^H)}{w^r(e^F - \tau)}}. 
\tag{29}
\]

By construction, because the supranational authority’s utility is continuous and increasing in the home country public good, and the public debt is positive at \( \eta^2 \) and zero at \( \eta^3 \), it must be that \( \eta^2 < \eta^3 \).

**Part 4**

In terms of magnitude of \( \eta^1 \), from (26), notice that if \( g^{H0} \simeq e^F - \tau Rr \), then \( \eta^1 \simeq \frac{1}{2} \). If \( w^r(g^{H0})/w^r(e^F - \tau R) \simeq k \), then \( \eta^1 \simeq \frac{1}{1+k} \), \( k \in \mathbb{R} \). For logarithmic utilities, if \( w^r(g^H) = \frac{1}{g^r} \) and \( g^{H0} \simeq \frac{e^F - \tau R}{k} \), then \( \eta^1 \simeq \frac{1}{1+k} \), \( k \in \mathbb{R} \). The same holds for \( \eta^2 \), from (29).

**A.2.2 Proof of Lemma 2**

From (26) and (29), the condition that \( \eta^1 < \eta^2 \) is equivalent to

\[
\frac{w^r(g^H)}{w^r(e^F - \tau)} < \frac{1}{1 + \beta} \frac{w^r(g^{H0})}{w^r(e^F - \theta(\omega^H + \omega^F) + x^0)}. 
\tag{30}
\]

Condition (30) can be used to derive the minimum value of \( g^H \) for each \( \tau \) such that \( \eta^1 < \eta^2 \). The value of \( \frac{w^r(g^H)}{w^r(e^F - \tau)} \) is decreasing in \( g^H - \tau \), or increasing in \( r + x \). So, for a given \( x \) (that can be obtained as function of \( e^F \)), the transfer \( \tau \) is higher for a higher \( r \), and the value of \( r \) increases as \( |v''(r)| / |u^{Hr}(x, \theta)| \) is smaller. This follows from the first-order conditions to the politician’s problem,

\[
\gamma v'(r) = (1 - \gamma) u^{Hr}(x, \theta),
\]
and from the intervention constraint

\[ r + x \leq \bar{x}. \]

These two expressions imply that

\[ \gamma v'(r) = (1 - \gamma)u^H(\bar{x} - r, \theta). \]

Then,

\[ \frac{\partial r}{\partial x} = \frac{1}{(1 - \gamma)u^H(\bar{x} - r, \theta) + 1}, \]

so any increase in \( x \) results in a larger relative increase the value of \( r \) as \( |v''(r)|/|u^H(x, \theta)| \) is smaller; therefore, \( \exists \bar{\rho} \) so that if \( |v''(r)|/|u^H(x, \theta)| < \bar{\rho} \), then \( r \) (and implicitly \( \tau \)) is sufficiently large for condition (30) to be satisfied when the equilibrium recapitalizations level is \( x \).

Given condition (27), \( \frac{\partial \rho}{\partial x} \) must be large enough such that \( \eta^1 < \eta^3 \). Then, \( \exists \bar{\rho} \) such that for \( |v''(r)|/|u^H(x, \theta)| > \bar{\rho} \), condition (27) is satisfied.

Finally, the value of cutoff \( \rho \) decreases as \( e^F \) increases, while the value of \( \bar{\rho} \) increases in the the size of \( e^F \); therefore, \( \exists \rho^* \) at which \( \rho = \bar{\rho} \), such that \( e^F > \rho^*, \rho < \bar{\rho} \). Then, for \( |v''(r)|/|u^H(x, \theta)| \in [\rho, \bar{\rho}] \), conditions (27) and (30) are satisfied.

\[ \text{A.2.3 Proof of Proposition 2} \]

For \( \eta < \eta^2 \), the participation constraint for the home government binds in equilibrium, positive transfers \( \tau \) are made, and \( \bar{x} \geq x^0 + r^0 \). The binding participation constraint implies

\[
(1 - \gamma)v(r) + \gamma w(g^H) + \gamma u(R(1 - \theta)\omega^H + Rx) + \beta \gamma w(g_1^H) = \\
(1 - \gamma)v(r^0) + \gamma w(g^{H0}) + \gamma u(R(1 - \theta)\omega^H + Rx^0) + \beta \gamma w(g_1^{H0}). (31)
\]

The electoral constraint is

\[ u^H(x, \theta) + w(g^H) \geq Z(b_1). \]
Then, constraint (31) can be reduced to

\[ (1 - \gamma)v(r) + Z(b_1) + \beta \gamma w(e^H - b_1) = (1 - \gamma)v(r^0) + Z(b_1^0) + \beta \gamma w(e^H - b_1^0). \] (33)

The policies \( r, x, g^H \) and \( b_1 \) are given by the politician’s first-order conditions (assuming an interior solution):

\[
\begin{align*}
\lambda - \xi &= (1 - \gamma)v'(r), \quad (34a) \\
\lambda - \xi &= \gamma (1 + \nu) \frac{\partial u^H(x, \theta)}{\partial x}, \quad (34b) \\
\lambda &= \gamma (1 + \nu) w'(g^H), \quad (34c) \\
\lambda &= \gamma w'(e^H - b_1) + \frac{\nu}{\beta} \gamma Z'(b_1), \quad (34d)
\end{align*}
\]

where \( \lambda, \xi, \) and \( \nu \) are the Lagrange multipliers on constraints (3a), (3b), and (7), respectively.

Consider the case in which a policy \((\tau, \overline{x})\) is implemented such that (31) holds with equality \( \forall \overline{x} \leq \theta(\omega^H + \omega^F) \). Let \( \{r, x, g^H, b_1\} \) denote the politician’s choices without the electoral constraint (when \( \nu = 0 \)), given \((\tau, \overline{x})\) and first-order conditions (34a)-(34d). Then, consider the case in which the electoral constraint is binding, and given the same \( \tau \) as before, the supranational authority sets \( \overline{x}^e \equiv \overline{x}(\chi = 1) \) such that \( x^e \equiv x(x^e) = x \) (same \( x \) as in the no elections case). Then, let \( \{r^e, x^e, g^{He}, b_1^e\} \) denote the policies chosen given \((\tau, \overline{x}^e)\) and first-order conditions (34a)-(34d), with \( \nu > 0, \xi > 0 \). Similarly, let \( \{r^{0e}, x^{0e}, g^{He}, b_1^{0e}\} \) denote the policies chosen in autarky, given first-order conditions (34a)-(34d), when \( \nu > 0, \xi = 0 \).

For simplicity, consider first the case when \( Z(b_1) = z = const \). We can show that if the results hold for this case given functions \( v(r) \) and \( u^H(x, \theta) \), then they also hold for the case when \( Z'(b_1) > 0 \) with the same functions \( v(r) \) and \( u^H(x, \theta) \).

The analysis compares the \( \chi = 0 \) and \( \chi = 1 \) cases under the following conditions:

- same transfer \( \tau \) in both cases;
• same recapitalizations in both case: \( x = x^e \), which implies \( x^e < x \) since \( \chi = 1 \) leads to an increase in \( x \);

• same electoral constraint \( Z(b_1) = Z(b_0^e) \) with and without the partial banking union.

Under this setup (with \( \varrho = 1 \) denoting participation in the banking union and \( \varrho = 0 \) denoting the autarky case), we examine the following problem: with the same "endowment" \( e + 1_{\varrho=1} \tau \), the politician maximizes utility given the budget constraint, the intervention constraint

\[
r + x \leq 1_{\varrho=1} [1_{\chi=0}x + 1_{\chi=1}x^e]
\]

and electoral constraint

\[
u^H(x, \theta) + w^H(b^e_1) \geq 1_{\chi=0}0 + 1_{\chi=1}Z(b_1).
\]

We start from the baseline case where constraint (33) binds in the \( \varrho = 0 \), \( \chi = 0 \) case.

If \( \chi = 1 \) and we consider the case when \( x(\chi = 1) = x^e = \bar{x} \) (i.e., only the electoral constraint changes), then from the politician’s constrained maximization problem it follows that at the politician’s constrained optimal allocation:

\[
(1 - \gamma) v(r^{e^*}) + \gamma Z(b_1^{e^*}) + \beta \gamma w(e^H - b_1^{e^*}) < (1 - \gamma) v(r^{0e}) + \gamma Z(b_1^{0e}) + \beta \gamma w(e^H - b_1^{0e}).
\]

Define

\[
\Delta V^{e^*} \equiv (1 - \gamma) v(r^{0e}) + \gamma Z(b_1^{0e}) + \beta \gamma w(e^H - b_1^{0e})
- \left[ (1 - \gamma) v(r^{e^*}) + \gamma Z(b_1^{e^*}) + \beta \gamma w(e^H - b_1^{e^*}) \right].
\]

Since \( x^e < \bar{x} \), and both constraints are binding, it follows that

\[
(1 - \gamma) v(r^e) + \gamma Z(b_1^e) + \beta \gamma w(e^H - b_1^e) > (1 - \gamma) v(r^{e^*}) + \gamma Z(b_1^{e^*})
+ \beta \gamma w(e^H - b_1^{e^*}).
\]
Define
\[
\Delta V^e \equiv (1 - \gamma)v(r^e) + \gamma Z(b^e_1) + \beta \gamma w(e^H - b^e_1) \\
- [(1 - \gamma)v(r^{e*}) + \gamma Z(b^{e*}_1) + \beta \gamma w(e^H - b^{e*}_1)].
\]

Then, the politician’s participation constraint does not hold at \((\tau, z^e)\) iff
\[
\Delta V^{e*} > \Delta V^e. \tag{37}
\]

Therefore, we need to derive the conditions under which the relaxation of constraint \((35)\), i.e., the decrease in the constraint by \(z^e\), is not sufficient to offset the decrease in utility due to constraint \((36)\). Constraint \((35)\) is relaxed and \(\Delta V^e\) increases as \(z^e\) decreases relative to \(z\). Therefore, let \(\varepsilon^*\) denote the value of \(z - z^e\) at which \(\Delta V^{e*} = \Delta V^e\). Then, \((37)\) holds for \(z - z^e < \varepsilon^*\).

The difference \(z - z^e = x + r - x^e - r^e = r - r^e\), since \(x = x^e\). Then, \((37)\) holds iff \(r - r^e < \varepsilon^*\). From the first-order conditions,
\[
(1 - \gamma)v'(r^e) = \gamma(1 + \nu) \partial u^H(x, \theta), \\
(1 - \gamma)v'(r) = \gamma \partial u^H(x, \theta),
\]
so
\[
v'(r^e) = (1 + \nu) v'(r),
\]
and substituting for \(\nu\):
\[
v'(r^e) = \frac{w'(g^H_1)}{w'(g^H)} v'(r)
\]

Therefore, \(r - r^e\) decreases as \(|v''(r)|\) increases relative to \(|w''(g^H)|\). For any value \(\varepsilon^*\), \(\exists |w''(g^H)|\) such that \(r - r^e < \varepsilon^*\), since if \(|w''(g^H)|\rightarrow 0\), then \(\frac{w'(g^H_1)}{w'(g^H)} \rightarrow 1\) and \(r^e \rightarrow r\) Then, for \(|v''(r)| / |w''(g^H)|\) sufficiently large, condition \((37)\) holds and a higher transfer \(\tau\) is needed in order to obtain \(x\) when \(\chi = 1\) and satisfy the politician’s participation constraint. The utility of donor households then decreases at each \(x\), since \(u^F(x, \theta) + w(e^F - \tau)\) decreases in \(\tau\).
A.2.4 Proof of Proposition 4

Denote by \{x^0, g^{H_0}, r^0, b^0\} the politician’s solution in autarky, obtained from maximizing (1) subject to \(\tau = 0\), (3a), (3c), (3d) and (7). Consider the problem for the supranational authority. Denote by \{\tau, x, b^{FR}_1\} the policies set by the supranational authority, and by \{x, g^H, r\} the policies resulting from the politician’s optimization problem under the banking union. For \(\eta < \eta^2\), the participation constraint for the politician binds, so

\[
(1 - \gamma)v(r) + \gamma w(g^H) + \gamma u^H(x, \theta) + \beta \gamma w(e^H - b^{FR}_1) = \\
(1 - \gamma)v(r^0) + \gamma w(g^{H_0}) + \gamma u^H(x^0, \theta) + \beta \gamma w(e^H - b^0_1). 
\] (38)

Moreover, the electoral constraint requires

\[
w(g^H) + u^H(x, \theta) = Z(b^{FR}_1), \\
w(g^{H_0}) + u^H(x^0, \theta) = Z(b^0_1).
\]

Given the electoral constraint, and (8b)-(8d), the supranational authority can set the value of \(r\) as a residual, since \((r, x, g^H, g^H_1)\) is the solution to a system of 4 equations with 4 unknowns.

Claim 1 \(r \leq r^0\).

Proof. Assume \(r > r^0\). Then, given the politician’s participation constraint, \(b^{FR}_1 > b^0_1\).

Consider first the case when \(Z(b) = z = \text{const}\). The supranational authority can decrease \(r\) by \(\Delta r\) and decrease \(b^{FR}_1\) by \(\frac{1}{\beta} \Delta b_1\). The change in the politician’s utility is approximated by

\[
\Delta P \simeq -(1 - \gamma)v'(r)\Delta r + \beta \gamma w'(e^H - b^{FR}_1) \frac{\Delta b_1}{\beta}.
\]

From the first-order conditions to the politician’s problem:\(^{21}\) \((1 - \gamma)v'(r^0) = \lambda = \gamma w'(e^H - b^0_1)\). So \((1 - \gamma)v'(r) < (1 - \gamma)v'(r^0) = \lambda\) and \(\lambda = \gamma w'(e^H - b^0_1) < \lambda = \gamma w'(e^H - b^0_1)\).

\(^{21}\)See the proof to Proposition 2.
\[ \gamma w'(e^H - b_1^{FR}), \] given the concavity of the utility functions. Then, imposing \( \Delta P = 0 \), it follows that \( \frac{\Delta b_1}{\beta} < \Delta r \) and the supranational authority can decrease the transfer \( \tau \) by \( \left( \Delta r - \frac{\Delta b_1}{\beta} \right) \) and increase its utility (since \( x \) weakly increases as \( b_1 \) decreases). Therefore, \( r > r^0 \) would not be optimal.

If \( Z'(b) > 0 \), then a decrease in \( b_1^{FR} \) also decreases household utility in period 0 by \( Z'(b) \frac{\Delta b_1}{\beta} \). This implies \( x + g^H \) may decrease. Denote then by \( \phi \Delta b_1, \phi \in (0,1) \), the decrease in \( r \) caused in equilibrium by the decrease of \( \frac{1}{\beta} \Delta b_1 \) in \( b_1^{FR} \). Then,

\[
\Delta P \approx -(1 - \gamma)\nu'(r)\phi \Delta b_1 + \gamma w'(e^H - b_1^{FR})\phi \Delta b_1 - \gamma Z'(b_1^{FR}) \frac{\Delta b_1}{\beta} + \gamma w'(e^H - b_1^{FR})(1 - \phi)\Delta b_1.
\]

From the first-order conditions to the politician’s problem, \( (1 - \gamma)\nu'(r) < \gamma w'(e^H - b_1^{FR}) \) and, from the assumption about \( Z'(b_1^{FR}) < w'(g^H) + u^H(x, \theta) \) (as discussed in the text), it follows that \( \frac\gamma{\beta} Z'(b_1^{FR}) < \gamma w'(e^H - b_1^{FR}) \). So, \( \exists \phi > 0 \) such that \( \Delta P > 0 \). Then, the supranational authority can decrease the transfer \( \tau \) and increase its utility (since \( x \) weakly increases as \( b_1 \) decreases). Therefore, \( r > r^0 \) would not be optimal. \( \blacksquare \)

Since \( r \leq r^0 \), it follows from the politician’s participation constraint that home household utility must be weakly higher under the banking union. The banking union always increases donor household utility, given the setup of the problem. Therefore, a Pareto improvement is achieved over autarky.

### A.2.5 Proof of Lemma 3

Below, I derive the conditions on the equilibrium policy functions \( \tau(b, \chi, \theta) \) and \( g(b, \chi, \theta) \) under which \( V(b, \chi, \theta) \) is concave and differentiable for \( b \in (b, \bar{b}) \). I then derive the conditions on \( b'(\tau, g|\chi, \theta) \) under which concavity and differentiability of \( V \) implies concavity and differentiability of \( S \). Finally, I derive the conditions on the utility functions that allow for these properties of the policy functions. This shows that an equilibrium can exist in which the value functions are concave and differentiable.
Part 1

A feasible set \( \{ \tau, \underline{x}, b, b' \} \) given \( \theta \) and \( \chi \) is an allocation that satisfies the conditions that \( \underline{x} \leq e^H - b + \tau + \beta b' \) and \( b' \in [\underline{b}, \bar{b}] \). A feasible set of current-period policies for the politician \( \{ r^f, g^{Hf}, x^f \} \) associated with an allocation \( \{ \tau, \underline{x}, b, b' \} \) and \( \{ \chi, \theta \} \) must satisfy the budget constraint, the intervention constraint, the electoral constraint and the constraints imposed by the exogenous upper/lower bounds on policies:

\begin{align*}
    r^f + x^f + g^{Hf} & \leq e^H - b + \tau + \beta b', \\
    r^f + x^f & \geq \underline{x}, \\
    w(g^{Hf}) + u^H(x^f, \theta) & \geq \chi Z(b, b'), \\
    x^f & \leq \theta (\omega^H + \omega^F), \\
    g^{Hf} & \geq 0, x^f \geq 0, r^f \geq 0.
\end{align*}

Let \( \Lambda(\tau, \underline{x}, b, b', \chi, \theta) \) denote the set of feasible current-period policies given \( \{ \tau, \underline{x}, b, b', \chi, \theta \} \). and \( \{ r, g^H, x \} \in \Lambda \) be the solution to the intra-period maximization problem faced by the politician. Let \( r^0 \) and \( x^0 \) be the policies chosen by the politician without constraint (39b) and with \( \tau = 0 \). Then \( r^0 + x^0 \leq \underline{x} \), because it is a weakly dominated strategy for the supranational authority to set the intervention bound \( \underline{x} \) to at least what the politician would choose without the bound. Therefore, constraint (39b) holds with equality in equilibrium. Then, \( g^H = e^H - b + \tau - \underline{x} + \beta b' \), so \( g^H \) is a concave function of debt \( b \) if \( \tau - \underline{x} \) is also a concave function of debt \( b \). Then, \( w(g^H(b, b')) \) is a concave, non-decreasing function of a concave function, and therefore it is also a concave function of debt.

**Condition 1** The function \( \tau - \underline{x} \) is a concave of debt \( b \).

- if \( \chi = 0 \), \( \gamma u^{H'}(x, \theta) = (1 - \gamma) v'(x, \theta) \), and \( u^H(x, \theta) + v(r) \) is a concave function of \( \underline{x} \), since the equilibrium conditions of the politician’s problem
require $\frac{\partial^2 x}{\partial z^2} + \frac{\partial^2 x}{\partial z^2} = 0$, and

$$\frac{\partial^2}{\partial x^2} [\gamma u^H(x, \theta) + (1 - \gamma) v(r)] = \gamma \frac{\partial^2 u^H(x, \theta)}{\partial x^2} \left( \frac{\partial x}{\partial x} \right)^2 + \gamma \frac{\partial u^H(x, \theta)}{\partial x} \frac{\partial^2 x}{\partial z^2} + (1 - \gamma) v''(r) \left( \frac{\partial r}{\partial x} \right)^2 + (1 - \gamma) v'(r) \frac{\partial^2 r}{\partial x^2},$$

(40)

which given the concavity of $u^H(x, \theta)$ and $v(r)$ implies

$$\frac{\partial^2}{\partial x^2} [\gamma u^H(x, \theta) + (1 - \gamma) v(r)] < 0.$$

Then, if $x(b, b')$ is concave and increasing, $u^H(x, \theta) + v(r)$ is also concave.

**Condition 2** The policy function $x(b, b')$ is concave.

- if $\chi = 1$, then $u^H(x, \theta) + w(g^H) = Z(b, b')$, and we can show that there exist functions $v(r)$, $u^H(x, \theta)$, and $w(g^H)$ such that that $(1 - \gamma)v(r) + \gamma u^H(x, \theta) + \gamma w(g^H)$ is weakly concave given $x(b, b')$ concave. The function $Z(b, b')$ is concave, so

$$\frac{\partial}{\partial b} [\gamma w(g^H) + \gamma u^H(x, \theta) + (1 - \gamma) v(r)] = \gamma \frac{\partial Z(b, b')}{\partial b} + (1 - \gamma) v'(r) \frac{\partial r}{\partial b},$$

$$\frac{\partial^2}{\partial b^2} [\gamma w(g^H) + \gamma u^H(x, \theta) + (1 - \gamma) v(r)] = \gamma \frac{\partial Z^2(b, b')}{\partial b^2} + (1 - \gamma) v''(r) \left( \frac{\partial r}{\partial b} \right)^2 + (1 - \gamma) v'(r) \frac{\partial^2 r}{\partial b^2}. $$


Then, \[ \frac{\partial^2}{\partial b^2} [\gamma w(g^H)\gamma u^H(x, \theta)+(1-\gamma)v(r)] \leq 0 \iff \]
\[ \gamma \frac{\partial Z^2(b, b')}{\partial b'^2} + (1 - \gamma) v''(r) \left( \frac{\partial r}{\partial b} \right)^2 + (1 - \gamma) v'(r) \frac{\partial^2 r}{\partial b'^2} \leq 0, \]
or
\[ \frac{\partial^2 r}{\partial b'^2} \leq -\frac{\gamma \frac{\partial Z^2(b, b')}{\partial b^2} + (1 - \gamma) v''(r) \left( \frac{\partial r}{\partial b} \right)^2}{(1 - \gamma) v'(r)}. \] (41)

If \( \bar{z}(b, b') \) is concave, then \( r(b) + x(b) \) must also be concave given constraint (39b). Then, condition (41) is immediately satisfied for \( \frac{\partial^2 r}{\partial b'^2} \) weakly concave, or for a sufficiently high \( v''(r) - u^{H''}(x, \theta) \), where \( u^{H''}(x, \theta) \) is the third derivative with respect to \( x \). For example, condition (41) is satisfied with \( \frac{\partial^2 r}{\partial b'^2} \) weakly concave if we assume \( v(r) \) to be an affine transformation of \( u^H(x, \theta) \).

Then, the utility of the politician in the current period is concave if \( \tau - \bar{z} \) and \( \bar{z}(b, b') \) are concave functions of \( b \), and the functions \( v(r), u^H(x, \theta) \), and \( w(g^H) \) satisfy the restrictions of condition (41).

Part 2

a) Concavity of the value function:

Assuming concavity of \( E[V(b', \chi', \theta', \tau'(b', \chi', \theta'), \bar{z}(b', \chi', \theta'))] \), the concavity of \( V(b, \chi, \theta, \tau, \bar{z}) \) can be shown by induction.

Consider two feasible values \( b_1, b_2 \in [b, \bar{b}] \), and \( b_3 = \theta b_1 + (1 - \theta) b_2, \) \( \theta \in (0, 1) \). Then, the supranational policies are given by functions \( \tau_1 = \tau(b_1, \chi, \theta), \ x_1 = \bar{z}(b_1, \chi, \theta), \ \tau_2 = \tau(b_2, \chi, \theta), \ x_2 = \bar{z}(b_2, \chi, \theta), \ \tau_3 = \tau(b_3, \chi, \theta), \ x_3 = \bar{z}(b_3, \chi, \theta). \) Let

\[ \{ x_1, r_1, g_1, b'_1 \} = \arg\max V(b_1, \chi, \theta, \tau_1, x_1), \]
\[ \{ x_2, r_2, g_2, b'_2 \} = \arg\max V(b_2, \chi, \theta, \tau_2, x_2). \]

Let \( b'_3 = \theta b'_1 + (1 - \theta) b'_2 \), and \( \{ x_3, r_3, g_3 \} = \arg\max (1 - \gamma) v(r) + \gamma u^H(x, \theta) + \gamma w(g^H), \) subject to constraints (39a)-(39e), given \( b_3, b'_3, \tau_3, \bar{z}_3. \)

Value \( b'_3 \) is feasible given that the set \( \Gamma(b|\chi, \theta) = [b, \bar{b}] \) is compact, and \( \{ x_3, r_3, g_3 \} \) is feasible given the above maximization problem. Since
\( u^P(b, b', \chi, \theta) \equiv \gamma w(g^H) + \gamma u^H(x, \theta) + (1 - \gamma) v(r) \) is concave under the assumptions from Part 1:

\[
\begin{align*}
V(b_3, \chi, \theta) & \geq u^P(b_3, b'_3, \chi, \theta) + \beta \mathbb{E}[V(b'_3)] \\
& \geq \vartheta [u^P(b_1, b'_1, \chi, \theta) + \beta \mathbb{E}[V(b'_1)]] \\
& \quad + (1 - \vartheta) [u^P(b_2, b'_2, \chi, \theta) + \beta \mathbb{E}[V(b'_2)]] .
\end{align*}
\]

By induction, the value function \( V(b_3, \chi, \theta) \) is therefore concave.

b) Differentiability of the politician’s value function:

The policy function is continuous, given the compact set \( \Gamma(b | \chi, \theta) \). The implicit utility function

\[
u^P(b, b', \chi, \theta) = (1 - \gamma)v(r(b, b', \chi, \theta)) + \gamma u^H(x(b, b', \chi, \theta), \theta) \]

\[
\quad + \gamma w(g^H(b, b', \chi, \theta))
\]

is concave and differentiable in \( b \). It then follows by Lemma 1 of Benveniste & Scheinkman (1979) that \( V(b, \chi, \theta, \tau, \underline{x}) \) is differentiable with respect to \( b \) over \((\underline{b}, \bar{b})\).

**Part 3**

Consider now the value function for the supranational authority. Denote the instantaneous utility function for the supranational authority as

\[
u^S(b, b', \chi, \theta, \tau, \underline{x}) \equiv \eta u^H(x(b, b', \chi, \theta, \tau, \underline{x}), \theta) + \eta w(g^H(b, b', \chi, \theta, \tau, \underline{x}))
\]

\[
\quad + (1 - \eta) u^F(x(b, b', \chi, \theta, \tau, \underline{x}), \theta) + (1 - \eta) w(e^F - \tau).
\]

Given Condition \(2\), a sufficient condition for \( \eta u^H(x, \theta) + (1 - \eta) u^F(x, \theta) \) to be a concave function of debt is that \( x(\underline{x}) \) is weakly concave. As above, we can show that there exist functions \( v(r) \), \( u^H(x, \theta) \), and \( w(g^H) \) under which this condition is satisfied along with condition (41). For example, both conditions are satisfied if we assume \( v(r) \) to be an affine transformation of \( u^H(x, \theta) \). Also,
the maximization problem for the supranational authority is a concave function of \( \tau \), so a sufficient condition for concavity with respect to debt is for \( \tau(b, b') \) to be concave. Then, \( u^S(b, b') \) is concave.

**Condition 3** The function \( \tau(b, b') \) is a concave function of \( b \).

**Part 4**

Consider feasible values \( \{ b_1, \tau_1, \underline{z}_1 \} \) and \( \{ b_2, \tau_2, \underline{z}_2 \} \). Let \( \{ b_3, \tau_3, \underline{z}_3 \} = \vartheta \{ b_1, \tau_1, \underline{z}_1 \} + (1 - \vartheta) \{ b_2, \tau_2, \underline{z}_2 \}, \forall \vartheta \in (0, 1) \). Then, \( \{ b_3, \tau_3, \underline{z}_3 \} \) is feasible and satisfies all constraints. Due to the concavity of \( u^S(b, b', \chi, \theta, \tau(b, b'), \underline{z}(b, b')) \), the concavity of \( S(b, \chi, \theta) \) follows by induction, analogous to the proof in Part 2: \( S(b_3, \chi, \theta) \geq \vartheta S(b_1, \chi, \theta) + (1 - \vartheta)S(b_2, \chi, \theta) \). Therefore, \( S(b, \chi, \theta) \) is concave.

**Part 5**

Consider the sequence of feasible values \( b^j \) such that \( b^j \to b \); then there is also a corresponding sequence \( \{ \tau^j, \underline{z}^j \} \) which converges to \( \{ \tau, \underline{z} \} \), since the instantaneous utility \( u^S(b, b', \chi, \theta, \tau(b, b'), \underline{z}(b, b')) \) is continuous in \( \{ \tau, \underline{z} \} \). Given the policy correspondence \( G(b^j, \tau^j, \underline{z}^j) \), we want to show that if \( b^{j'} \in G(b^j, \tau^j, \underline{z}^j) \), then \( \exists \) a convergent subsequence \( b^{j''} \to b' \) with \( b' \in G(b, \tau, \underline{z}) \). Since \( \{ \tau^j, \underline{z}^j \} \) are defined over compact sets, \( \{ \tau, \underline{z} \} \) is feasible. Moreover, it implies a convergent subsequence \( \{ b^{j''} \} \) must exist. Then, by the Dominated Convergence Theorem, \( b' = G(b, \tau, \underline{z}) \). Therefore, the policy function is continuous.

**Part 6**

Consider the sequence \( \{ \tau, \underline{z} \} \) associated with the debt \( b \in (\underline{b}, \bar{b}) \). Then, with \( S(b, \chi, \theta) \) concave and a continuous policy function, by the argument of Lemma 1 in Benveniste & Scheinkman (1979), \( S(\cdot) \) is differentiable in \( b \) over \( (\underline{b}, \bar{b}) \).

**Part 7**

We now show that Conditions 1, 2, and 3 can be satisfied in equilibrium. First, from the politician’s problem, \( b' \) is a decreasing function of \( \tau - \underline{z} \), which, using the inverse function properties, means that \( \tau - \underline{z} \) being concave (Condition 1) requires that \( b'(\tau - \underline{z}) \) is a concave function of \( \tau - \underline{z} \). This then implies that \( g^H(\tau - \underline{z}) \) is also concave, given the politician’s budget constraint and
are chosen by the politician according to the first-order conditions. Moreover, since \( b' \) is a decreasing function of \( \tau - \overline{x} \), if \( b'(\tau - \overline{x}) \) is a concave function of \( \tau - \overline{x} \), then \( \tau - \overline{x}(\tau) \) must be a convex function of \( \tau \), which requires that \( \overline{x}(\tau) \) is concave.

In the case considered, the participation constraint for the home government binds in equilibrium, so \( \forall i \in \{1, 2, 3\} : \)

\[
(1 - \gamma) v(r_i) + \gamma u^H(x_i, \theta) + \gamma w(g_i^H) + \beta E V(b_i', \chi', \theta', \tau'(b_i', \chi', \theta'), \overline{x}(b_i', \chi', \theta')) = \\
(1 - \gamma) v(r_0) + \gamma u^H(x_0, \theta) + \gamma w(g_0^H) + \beta E V(b_0', \chi', \theta', \tau'(b_0', \chi', \theta'), \overline{x}(b_0', \chi', \theta')). \tag{42}
\]

Condition (42) allows us to derive \( \overline{x}(\tau) \) given that the policies \( \{r_i, x_i, g_i^H, b_i'\} \) are chosen by the politician according to first-order conditions (34a)-(34d).

Let \( \tau_3 = \alpha \tau_1 + (1 - \alpha) \tau_2 \), and consider the case in which \( b'(\tau, \overline{x}(\tau)) \) is concave. Then, \( b_3' \geq \alpha b_1' + (1 - \alpha) b_2' \), where \( b_i' \equiv b'(\tau_i), \forall i = 1, 2, 3. \) Since \( E [V(b')] \) is decreasing in \( b' \), \( E [V(b_3')] < E [V(\alpha b_1' + (1 - \alpha) b_2')] \) and since it is concave, \( E [V'(b_3')] < E [V'(\alpha b_1' + (1 - \alpha) b_2')] \).

Changes in \( \tau \) in the current period do not change the outside option of the politician, so condition (42) implies:

\[
(1 - \gamma) v(r_3) + \gamma u^H(x_3, \theta) + \gamma w(g_3^H) + \beta E [V(b_3')] = \\
\alpha [(1 - \gamma) v(r_1) + \gamma u^H(x_1, \theta) + \gamma w(g_1^H) + \beta E [V(b_1')]] \\
+ (1 - \alpha) [(1 - \gamma) v(r_2) + \gamma u^H(x_2, \theta) + \gamma w(g_2^H) + \beta E [V(b_2)]] \tag{43}
\]

with \( x_i \equiv x(\tau_i), r_i = r(\tau_i), g_i^H = g(\tau_i), \forall i = 1, 2, 3. \)

When \( \chi = 0 \), from the first-order conditions, \( E [-V'(b_i')] = \gamma w'(g_i^H) \), so \( w'(g_3^H) > w'(\alpha g_1^H + (1 - \alpha) g_2^H) \). Since \( w(g^H) \) is a concave function, this requires \( g_3^H(\tau_3) < \alpha g_1^H(\tau_1) + (1 - \alpha) g_2^H(\tau_2) \), so \( g^H(\tau) \) is convex. Yet, given the budget constraint (10a), this implies:

\[
g^H(\tau) - \beta b'(\tau) = \tau - \overline{x}(\tau), \tag{44}
\]
so
\[
\frac{\partial^2 g^H(\tau)}{\partial^2 \tau} - \beta \frac{\partial^2 y(\tau)}{\partial^2 \tau} = - \frac{\partial^2 x(\tau)}{\partial \tau^2}.
\] (45)

Therefore,
\[
- \frac{\partial^2 x(\tau)}{\partial \tau^2} \geq 0,
\] (46)
and \(x(\tau)\) is concave. Then, from the supranational authority’s maximization problem, \(\tau(b)\) is the solution to the maximization of a concave function over a convex set, so \(\tau(b)\) is concave. Conditions 1, 2 and 3 follow.

When \(\chi = 1\), since \(Z(b, b')\) is concave and non-decreasing in \(b'\), it follows that \(Z(b, b_3) \geq \alpha Z(b, b'_1) + (1 - \alpha) Z(b, b'_2)\). From (43),
\[
(1 - \gamma) v(r_3) + \beta \mathbb{E}[V(b'_3)] \leq \alpha [(1 - \gamma) v(r_1) + \beta \mathbb{E}[V(b'_1)]) + (1 - \alpha) [(1 - \gamma) v(r_2) + \beta \mathbb{E}[V(b'_2)]).
\]

Then, a sufficient condition for \(x(\tau)\) to be concave is that \(g^H(\tau)\) is convex and \(x(\tau)\) is concave such that:
\[
u''(x, \theta) \left( \frac{\partial x}{\partial \tau} \right)^2 + w''(g^H) \left( \frac{\partial g^H}{\partial \tau} \right)^2 + u''(x, \theta) \frac{\partial^2 x}{\partial \tau^2} + w'(g^H) \frac{\partial^2 g^H}{\partial \tau^2} = Z''(b, b') \left( \frac{\partial b'}{\partial \tau} \right)^2 + Z'(b, b') \frac{\partial^2 b'}{\partial \tau^2},
\]
and \(r(\tau)\) either concave or \(v''(r) - u''(x, \theta)\) is positive and sufficiently high such that (34a) and (34b) are satisfied. Then,
\[
\frac{\partial^2 x}{\partial \tau^2} + \frac{\partial^2 \tau}{\partial \tau^2} = \frac{\partial^2 x}{\partial \tau^2} = - \frac{\partial^2 g^H(\tau)}{\partial \tau^2} + \beta \frac{\partial^2 y(\tau)}{\partial \tau^2},
\]
and (44)-(46) are satisfied; therefore, \(\tau(b)\) is the solution to the maximization of a concave function over a convex set, and Conditions 1, 2 and 3 follow. Finally, the condition that \(g^H(\tau)\) convex and \(b'(\tau)\) concave requires that
\[
w''(g^H) \left( g^H(\tau) \right)^2 + w''(g^H) g^{H^H}(\tau) = \mathbb{E}[-V''(b')] (b'(\tau))^2 + \mathbb{E}[-V''(b')] b''(\tau).
\]
The above conditions are satisfied, for example, whenever \( v(r), u^H(x, \theta), w(g^H) \) are affine transformations of each other. Thus, there exist concave utility functions such that \( V(b, \theta, \chi) \) and \( S(b, \theta, \chi) \) are concave and differentiable over \( b \in (\underline{b}, \bar{b}) \).

### A.2.6 Proof of Proposition 5

The participation constraint for the politician binds in equilibrium for all debt levels \( b \in [\underline{b}, \bar{b}] \), and the equilibrium rule \( x \) binds in every period. If \( \chi = 0 \), constraint (13) takes the form:

\[
(1 - \gamma) v(r_{01}) + \gamma u^H(x_{01}, \theta) + \gamma w(g_{01}^H) \\
+ \beta \mathbb{E} [V(b'_{01}, \chi', \theta', \tau'((b'_{01}, \chi', \theta'), \underline{\chi}'(b'_{01}, \chi', \theta')))] = (1 - \gamma) v(r_{00}) \\
+ \gamma u^H(x_{00}, \theta) + \gamma w(g_{00}^H) \\
+ \beta \mathbb{E} [V(b'_{00}, \chi', \theta', \tau'((b'_{00}, \chi', \theta'), \underline{\chi}'(b'_{00}, \chi', \theta')))],
\]

where the first subscript denotes \( \chi = 0 \) and the second whether \( \varrho = 1 \) or \( \varrho = 0 \). A binding rule \( r_{01} + x_{01} = \underline{x}_0 \) implies \( r_{01} > r_{00} \) and \( x_{01} > x_{00} \), given the politician’s first-order conditions.

When \( \chi = 1 \), the binding participation constraint (13) takes the form:

\[
(1 - \gamma) v(r_{11}) + \gamma u^H(x_{11}, \theta) + \gamma w(g_{11}^H) \\
+ \beta \mathbb{E} [V(b'_{11}, \chi', \theta', \tau'((b'_{11}, \chi', \theta'), \underline{\chi}'(b'_{11}, \chi', \theta')))] = (1 - \gamma) v(r_{10}) \\
+ \gamma u^H(x_{10}, \theta) + \gamma w(g_{10}^H) \\
+ \beta \mathbb{E} [V(b'_{10}, \chi', \theta', \tau'((b'_{10}, \chi', \theta'), \underline{\chi}'(b'_{10}, \chi', \theta')))],
\]

where the two subscripts denote that the electoral constraint binds and whether \( \varrho = 1 \) or \( \varrho = 0 \). A binding rule \( r_{11} + x_{11} = \underline{x}_1 \) implies \( r_{11} \geq r_{10} \) and \( x_{11} \geq x_{10} \).

Starting at time \( s \), the expected politician utility under a partial banking
union (given no deviation):

\[ V_s = \max_{\{r_t, g_t, x_t\}} \mathbb{E}_{\theta, \chi} \left\{ \sum_{t=s}^{\infty} \beta^{t-s} [(1 - \gamma)v(r_t) + \gamma w(g_t^H) + \gamma u^H(x_t, \theta)] \right\}, \quad (49) \]

and the politician utility under no partial banking union in any period is:

\[ V_s^0 = \max_{\{r_t, g_t, x_t\}} \mathbb{E}_{\theta, \chi} \left\{ \sum_{t=s}^{\infty} \beta^{t-s} [(1 - \gamma)v(r_t^0) + \gamma w(g_t^{H0}) + \gamma u^H(x_t^0, \theta_t)] \right\}. \quad (50) \]

In equilibrium, the agreement \((\tau, \underline{x})\) is offered such that the participation constraint for the politician binds in every period. So \(V_s = V_s^0:\)

\[ \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} [(1 - \gamma)v(r_t) + \gamma w(g_t^H) + \gamma u^H(x_t, \theta_t)] = \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s}[(1 - \gamma)v(r_t^0) + \gamma w(g_t^{H0}) + \gamma u^H(x_t^0, \theta_t)]. \quad (51) \]

Moreover, the intervention rule \(\underline{x}\) binds every period, so \(\forall t, r_t \geq r_t^0\) and \(v(r_t) \geq v(r_t^0)\). Then, from (51), \(\mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s}(1 - \gamma)v(r_t) \geq \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s}(1 - \gamma)v(r_t^0)\) implies

\[ \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} [w(g_t^H) + u^H(x_t, \theta_t)] \leq \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} [w(g_t^{H0}) + u^H(x_t^0, \theta_t)]. \quad (52) \]

**A.2.7 Proof of Proposition 6**

The supranational authority is choosing the transfer and intervention level \((\tau, \underline{x})\) every period in order to solve program (12) subject to the participation of both governments. Under parameters \(\eta\) and \(e^H\) that satisfy Assumption 4, the participation constraint for the politician binds in equilibrium for all debt levels \(b \in [\underline{b}, \bar{b}]\), and the equilibrium rule \(\underline{x}\) binds in every period. The
participation constraint (13) requires

\[
(1 - \gamma) v(r(x, \tau, \chi, \theta)) + \gamma u^H(x(x, \tau, \chi, \theta), \theta) + \gamma w(g^H(x, \tau, \chi, \theta)) + \\
\beta E[V(b'(x, \tau, \chi, \theta), \tau'(b', \chi', \theta'), x'(b', \chi', \theta'), \chi', \theta')] = \\
(1 - \gamma) v(r^0(\chi, \theta)) + \gamma u^H(x^0(\chi, \theta), \theta) + \gamma w(g^{H0}(\chi, \theta)) + \\
\beta E[V(b^0(\chi, \theta), \tau'(b^0, \chi', \theta'), x'(b^0, \chi', \theta'), \chi', \theta')].
\]

where

\[
u^H(x(x, \tau, \theta) + w(g^H(x, \tau)) = \chi Z(b, b'(x, \tau, \chi, \theta)),
\]

\[
u^H(x^0, \theta) + w(g^{H0}) = \chi Z(b, b^0(\chi, \theta)).
\]

Consider the case in which \(Z(b, b') = z = const\.) and assume a temporary marginal increase in \(Z(b, b')\) in the current period to \(\tilde{Z}(b, b') = Z(b, b') + \Delta Z\), for a small \(\Delta Z \rightarrow 0\). The value of the electoral constraint does not change in future periods. The case when the change is permanent is discussed below. As in the proof to Proposition 2, assume transfer \(\tau\) in the current period is kept constant, and \(x\) is decreased to \(x^z\), such that, given the politician’s problem, \(x(x^z) = x(x)\). Then, the effects of a change in \(Z(b, b')\) can be divided into two parts:

- an decrease in the politician’s utility due to the increase in \(Z(b, b')\) of \(\Delta Z\), keeping the current period supranational policies \((\tau, x)\) fixed;
- an increase in the politician’s utility due to the decrease in \(x\) by \(\Delta x\), to \(x^z\), such that, given the politician’s problem, \(x(x^z) = x(x)\).

Consider the first effect, due to the increase of \(\Delta Z\) in \(Z(b, b')\), and define \(\Delta x \equiv x(\tilde{Z}, x) - x(Z, x)\), and \(\Delta g^H, \Delta r, \Delta b'\) analogously. The electoral constraint implies

\[
u^H(x, \theta) \Delta x + w'(g^H) \Delta g^H \simeq \Delta Z,
\]

while the first-order conditions (16) and (17) to the politician’s problem lead

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to

\[ u^H(x, \theta) = \frac{\lambda - x}{\lambda} w'(g^H), \]

\[ u^H(x, \theta) = \frac{(1 - \gamma)v'(r)}{E[-V(b', r', x', \chi', \theta')]} w'(g^H). \]

Thus,

\[ u^{H\theta}(x, \theta) \Delta x \simeq \frac{(1 - \gamma)v'(r)}{E[-V(b', r', x', \chi', \theta')]} w''(g^H) \Delta g^H, \]

so

\[ \frac{\Delta x}{\Delta g^H} \simeq \frac{(1 - \gamma)v'(r)}{E[-V(b', r', x', \chi', \theta')]} u^{H\theta}(x, \theta), \]

and the above system of equations yields

\[ \Delta g^H \simeq \frac{\Delta Z}{w'(g^H) + u^H(x, \theta) E[-V(b', r', x', \chi', \theta')]} \frac{w''(g^H)}{u^{H\theta}(x, \theta)}, \]

\[ \Delta x \simeq \frac{\Delta Z}{w'(g^H)} \left( \frac{1}{E[-V(b', r', x', \chi', \theta')]} \frac{w''(g^H)}{u^{H\theta}(x, \theta)} \right)^{-1} + u^H(x, \theta). \]

(56)

Moreover, from the intervention constraint (10b), \( \Delta x = -\Delta r \), and from the budget constraint (10a), \( \Delta g^H = \beta \Delta b' \).

Then, the change in politician’s utility following the increase in \( Z(b, b') \) of \( \Delta Z \) is given by \( \Delta V^A / \Delta Z = -\nu \), where \( \nu \) is the Lagrange multiplier on the electoral constraint (10c), and it can be expressed as

\[ \Delta V^A = -(1 - \gamma)v'(r)\Delta x + \gamma u^H(x, \theta) \Delta x \]

\[ + \gamma w'(g^H) \Delta g^H - \beta E[-V'(b', r', z', \chi', \theta')] \Delta g^H, \]

so, defining \( \varsigma \equiv \frac{(1 - \gamma)v'(r)}{E[-V'(b', r', z', \chi', \theta')]} \frac{w''(g^H)}{u^{H\theta}(x, \theta)}, \)

\[ \frac{\Delta V^A}{\Delta Z} = \gamma - \frac{(1 - \gamma)v'(r)\varsigma + \beta E[-V'(b', r', z', \chi', \theta')]}{w'(g^H) + \varsigma u^H(x, \theta)}. \]

(57)

Consider now the second effect, that of decreasing \( x \) by \( \Delta x^z \) to \( \bar{x} \), and define \( \Delta x^z \equiv x(\bar{Z}, \bar{x}) - x(\bar{Z}, z^z), \) and \( \Delta g^{H^z}, \Delta r^z, \Delta b^z \) analogously. The
increase in the politician’s utility due to this change is approximated by $\Delta V^B / (-\Delta x^z) = \lambda$, where $\lambda$ is the Lagrange multiplier on the intervention constraint (10b). The intervention constraint implies $\Delta x^z + \Delta r^z = \Delta \bar{x}$, and the electoral constraint implies

$$u^{ll}(x(\bar{Z}, x), \theta)\Delta x^z + u'(g^l(\bar{Z}, x))\Delta g^H = 0.$$ 

Using the first-order conditions to the politician’s problem, this can be expressed as,

$$\Delta V^B = \beta \mathbb{E} [-V'(b', \tau', x', \chi', \theta')] \Delta b^z - v'(r)\Delta r^z$$

$$= \left[\beta \mathbb{E} [-V'(b', \tau', x', \chi', \theta')] - v'(r(\bar{Z}, \bar{x}))\right] \Delta \bar{x}, \quad (58)$$

since

$$\frac{u^{ll}(x(\bar{Z}, x), \theta)}{u'(g^l(\bar{Z}, x))} \beta \mathbb{E} [-V'(b', \tau', x', \chi', \theta')] = \lambda - \lambda = v'(r(\bar{Z}, \bar{x})).$$

From (57) and (58), the total decrease in the politician’s utility under the partial banking union with $x(\bar{x}) = x(x)$ is

$$\Delta V^A - \Delta V^B = \gamma \nu \Delta \bar{Z} - \lambda \Delta \bar{x}$$

$$= \left[\frac{(1 - \gamma) v'(r)}{\gamma u^{ll}(x, \theta)} - 1\right] \gamma \Delta \bar{Z}$$

$\quad - (\beta \mathbb{E} [-V'(b', \tau', x', \chi', \theta')] - (1 - \gamma) v'(r)) \Delta \bar{x}. \quad (59)$

Finally, $\Delta \bar{x}$ is determined such that $x(\bar{x}) = x(x)$, so $\Delta x^z = \Delta x$; from the first-order conditions (15) and (16), it follows that

$$(1 - \gamma) v'(r) = \gamma u^{ll}(x, \theta),$$

$$(1 - \gamma) v''(r)\Delta r^z = \gamma u^{lll}(x, \theta)\Delta x^z,$$

so

$$\Delta r^z = \frac{\gamma u^{lll}(x, \theta)}{(1 - \gamma) v''(r)} \Delta x, \quad (60)$$

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and, using (60) and (56) in the intervention constraint, we obtain

\[
\Delta z = \left[ \frac{\gamma u^{Hr}(x, \theta)}{(1 - \gamma) v'(r)} + 1 \right] \frac{\Delta Z}{w'(g^H)\zeta^{-1} + u^{Hr}(x, \theta)}.
\]

Then, (59) becomes

\[
\frac{\Delta V^A - \Delta V^B}{\Delta Z} = \left[ \frac{(1 - \gamma) v'(r)}{\gamma u^{Hr}(x, \theta)} - 1 \right] \gamma - (\beta \mathbb{E} [-V'(b', \tau', x', \chi')] \left[ \frac{\gamma u^{Hr}(x, \theta)}{(1 - \gamma) v'(r)} + 1 \right] - (1 - \gamma) v'(r) \frac{\gamma u^{Hr}(x^0, \theta)}{w'(g^H)\zeta^{-1} + u^{Hr}(x, \theta)}. \tag{61}
\]

Without the partial banking union, an increase in \( Z \) of \( \Delta Z \) would lead to a change in politician’s utility of

\[
\frac{\Delta V^{A0}}{\Delta Z} = \gamma v^0 \\
= \left[ \frac{(1 - \gamma) v'(r^0)}{\gamma u^{Hr}(x^0, \theta)} - 1 \right] \gamma. \tag{62}
\]

The participation constraint for the politician is not satisfied at \((\tau, x^z)\) if

\[
\Delta V^A - \Delta V^B > \Delta V^{A0},
\]

which using (61) and (62) means

\[
\left[ \frac{(1 - \gamma) v'(r)}{\gamma u^{Hr}(x, \theta)} - \frac{(1 - \gamma) v'(r^0)}{\gamma u^{Hr}(x^0, \theta)} \right] \gamma - (\beta \mathbb{E} [-V'(b', \tau', x', \chi')] \left[ \frac{\gamma u^{Hr}(x, \theta)}{(1 - \gamma) v'(r)} + 1 \right] - (1 - \gamma) v'(r) \frac{\gamma u^{Hr}(x^0, \theta)}{w'(g^H)\zeta^{-1} + u^{Hr}(x, \theta)} > 0. \tag{63}
\]

Since, in terms of Lagrange multipliers,

\[
\beta \mathbb{E} [-V'(b', \tau', x', \chi')] - (1 - \gamma) v'(r) = \kappa > 0,
\]

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and
\[
\frac{(1 - \gamma)v'(r)}{\gamma u^H(x, \theta)} - \frac{(1 - \gamma)v'(r^0)}{\gamma u^H(x^0, \theta)} = \nu - \nu^0 = \beta \mathbb{E}[V'(b', \tau', x', \chi', \theta')]
\]
\[
- \beta \mathbb{E}[-V'(b^0, \tau', x', \chi', \theta')]
\]
\[
> 0,
\]
it follows that the left-hand side of condition (63) is decreasing in the term
\[
\left[ \frac{\gamma u^H(x, \theta)}{(1 - \gamma)v''(r)} + 1 \right] / \left[ w'(g^H) \mathbb{E}[-V(b', \tau', x', \chi', \theta')] u^{H^H}(x, \theta) + u^{H^H}(x, \theta) \right].
\]
Therefore, the left-hand side of condition (63) is increasing in \(|v''(r)|\) and decreasing in \(|w''(g^H)|\). For any value of \(|w''(g^H)|\), let \(v^{**}\) denote the value of \(|v''(r)|\) at which condition (63) holds with equality. Then, for each \(|w''(g^H)|\), \(\forall |v''(r)| > v^{**}\), condition (63) is satisfied, which implies that the participation constraint on the politician is not satisfied. Then, transfers \(\tau\) must be increased in order to obtain the same level of recapitalizations \(x\) as before, so donor household utility is lower in the current period. Moreover, public debt \(b'\) increases following an increase in \(Z\), which implies lower expected utility for the donor households. These two effects together imply that donor household welfare decreases.

If the change in \(Z\) is permanent (\(Z\) decreases in future periods as well), then the continuation value of the politician decreases; due to concavity, the decrease is the continuation value is higher at \(b' > b^0\), and the same analysis as above goes through. The analysis is analogous for the case when \(\frac{\partial Z(b, \theta)}{\partial \theta} > 0\), which the change that the first-order condition on debt is changed from \(\beta \mathbb{E}[-V'(b', \tau', x', \chi', \theta')]\) to \(\beta \mathbb{E}[-V'(b', \tau', x', \chi', \theta')] + \nu \gamma \frac{\partial Z(b, \theta)}{\partial \theta}\).\(^{22}\)

\(^{22}\)Details available on request.
A.2.8 Proof of Lemma 4

The politician’s static choices of \( x(b, \chi, \theta, \tau, \phi, b') \), \( g(b, \chi, \theta, \tau, \phi, b') \), and \( r(b, \chi, \theta, \tau, \phi, b') \) lead to concave and differentiable functions \( u^H(\cdot) \), \( u^F(\cdot) \) and \( w(\cdot) \). Then, by induction, the value function \( S(\cdot) \) is concave. Moreover, the policy functions are continuous, and, by the standard arguments,\(^{23}\) \( S(b, \chi, \theta) \) is differentiable in \( b \) over \((b, \bar{b})\).

A.2.9 Proof of Proposition 7

As before, the participation constraints for the politician are given by (47) and (48). We will show that when \( \chi = 1 \), the equilibrium allocation has the property that rents are weakly lower than without the banking union \((r_{11} \leq r_{10})\).

Assume an allocation \( \{r_{11}, x_{11}, g_{11}, b'_{11}\} \) under the partial banking union. Consider first the case when \( Z(b, b') = z = \text{const.} \) and \( r_{11} > r_{10} \), where \( r_{10} \) is the choice of rents without the banking union in the current period. The participation constraint for the politician must hold with equality. Therefore, at \( \tau > 0, \phi_1 \) is binding, and \( b'_{11} > b'_{10} \). Assume a decrease by some small \( \beta \varepsilon \) of \( r_{11} \) and a decrease of \( \varepsilon \) in \( b'_{11} \), along with a decrease of \( \beta \varepsilon \) in \( \phi_1 \). Without the banking union, \((1 - \gamma)\nu'(r_{10}) = \mathbb{E} \left[ -\frac{\partial V(b_{10}, \phi', \theta', \tau', \phi_1, x_{11})}{\partial b_{10}} \right] \). With the banking union, \((1 - \gamma)\nu'(r_{11}) < \mathbb{E} \left[ -\frac{\partial V(b_{11}', \phi', \theta', \tau', \phi_1, x_{11})}{\partial b_{11}} \right] \), and the change would increase the utility of the politician. Moreover, it would not change the actual recapitalization level \( x_{11} \) (because \( \phi_1 \) is decreased by \( \beta \varepsilon \)). The supranational authority’s utility decreases with debt \( b' \), so a policy of decreasing \( b'_{11} \) and \( \phi_1 \) by \( \beta \varepsilon \) increases the utility of the supranational authority. Since \( r_{11} \) and \( b'_{11} \) were arbitrary, this argument holds for any allocation with \( r_{11} > r_{10} \) and \( b'_{11} > b'_{10} \) in which the participation constraint for the politician binds.

If \( r_{11} = r_{10} \) and \( b'_{11} < b'_{10} \), then the participation constraint is slack. Rents can be decreased by a small \( \varepsilon \) and transfers decreased by the same amount. This does not change the other policy choices, the participation constraint of the politician still holds, while the utility of the supranational authority increases.

\(^{23}\) Lemma 1 of (Benveniste & Scheinkman 1979).
increases. Therefore, the original allocation was not optimal.

If $\frac{\partial Z(b, \theta)}{\partial \theta} > 0$, then the above analysis is modified analogously to the proof of Proposition 4, and the property that $r_{11} \leq r_{10}$ follows.

A.2.10 Proof of Lemma 5

As in proof to Lemma 5, denote by $\{x^R, \tau^R\}$ the solution to problem (19) given the additional restriction that $\tau \geq x - (x^0 + r^0)$, i.e., the increase in interventions in the current period is fully financed by transfers. Also, $\{x^0, r^0, g^{H0}, b^0\}$ denote the politician’s policy choices under no banking union in the current period. The supranational authority’s problem (19) leads to the following first order condition for $\tau^R$:

$$[\eta u^H(x, \theta) + (1 - \eta) u^F(x, \theta)] \frac{\partial x}{\partial x^R} = (1 - \eta) w'(e^F - \tau^R).$$  \hspace{1cm} (64)

At this transfer level, the marginal cost of increasing $x^R$ without increasing $\tau^R$ is given by

$$-\eta w'(g^H) \frac{\partial g^{H0}}{\partial x^R} + \beta \mathbb{E} \left[ - \frac{\partial S'}{\partial b^0} \right] \frac{\partial b^0}{\partial x^R},$$

which, given the politician’s budget constraint (10a), can be re-written as

$$\eta w'(g^H) \left( - \frac{\partial g^{H0}}{\partial x^R} \right) + \beta \mathbb{E} \left[ - \frac{\partial S'}{\partial b^0} \right] \left( 1 + \frac{\partial g^{H0}}{\partial x^R} \right).$$

Therefore, the constraint $\tau^R \geq x^R - (x^0 + r^0)$ will not bind if

$$(1 - \eta) w'(e^F - \tau^R) - \eta w'(g^H) \left( - \frac{\partial g^{H0}}{\partial x^R} \right) - \beta \mathbb{E} \left[ - \frac{\partial S'}{\partial b^0} \right] \left( 1 + \frac{\partial g^{H0}}{\partial x^R} \right) < 0. \hspace{1cm} (65)$$

From (64), an increase in $\eta$ leads to a higher $\tau^R$ and a lower $(1 - \eta) w'(e^F - \tau^R)$. Then, the left-hand side of (65) is decreasing in $\eta$. Condition (65) holds at $\eta = 1$. Therefore, given the monotonicity of (65), $\exists \eta^{3d} \geq 0$ such that (65) holds with equality if $\eta^{3d} > 0$. By Assumption 4, condition (65) does not hold at $\eta = 0$, since otherwise the participation constraint for the politician would be slack. It follows that $\eta^{3d} > 0$ and $\eta^{3d} > \eta^{2d}$, the maximum value of $\eta$ under
Assumption 4.

Denote by \( \{ x^{Rr}, \tau^{Rr} \} \) the solution to problem (19) given the additional restrictions that, in the current period, \( \tau \geq x - (x^0 + r^0) \) and \( r \leq r^0 \). The supranational authority’s problem with these two additional constraints leads to the following first order condition for \( \tau^{Rr} \):

\[
[\eta u^H(x, \theta) + (1 - \eta) u^F(x, \theta)] = (1 - \eta) w'(e^F - \tau^{Rr}).
\]

At this transfer level, the marginal cost of increasing \( x^{Rr} \) without increasing \( \tau^{Rr} \) is given by

\[-\eta w'(g^H) \frac{\partial g^{H0}}{\partial x^{Rr}} + \beta \mathbb{E} \left[-\frac{\partial S'}{\partial b^{00}} \frac{\partial b^{00}}{\partial x^{Rr}}\right],\]

which, given the politician’s budget constraint (10a), can be re-written as

\[
\eta w'(g^H) \left(-\frac{\partial g^{H0}}{\partial x^{Rr}}\right) + \mathbb{E} \left[-\frac{\partial S'}{\partial b^{00}} \right] \left(1 + \frac{\partial g^{H0}}{\partial x^{Rr}}\right).
\]

Therefore, the constraint \( \tau^{Rr} \geq x^{Rr} - (x^0 + r^0) \) will not bind if

\[
(1 - \eta) w'(e^F - \tau^{Rr}) - \eta w'(g^H) \left(-\frac{\partial g^{H0}}{\partial x^{Rr}}\right) - \mathbb{E} \left[-\frac{\partial S'}{\partial b^{00}} \right] \left(1 + \frac{\partial g^{H0}}{\partial x^{Rr}}\right) < 0. \tag{66}
\]

Then, by the same argument as above, \( \exists \eta^{ld} \geq 0 \) such that (65) holds with equality if \( \eta^{ld} > 0 \). By Assumption 4, condition (66) does not hold at \( \eta = 0 \), since otherwise the participation constraint for the politician would be slack. Therefore, \( \eta^{ld} > 0 \).

A.2.11 Proof of Proposition 8

From Proposition 7, it follows that the supranational authority would not set debt higher than \( b'_{10} \). The supranational authority would prefer to set debt below \( b'_{10} \) whenever the marginal benefit of lower debt (in terms of future expected utility) is higher than the cost of additional transfers. This constraint is exactly \( \eta^{ld} < \eta^{2d} \), given the analysis from Lemma 5. Given that (13) is binding, this also implies that rents in period \( t \) are lower under the banking
union than under no banking union if there is electoral accountability in that period: \( r_{t,11} < r_{t,10} \).

A Pareto improvement can be achieved if the expected home household welfare is at least as high as under no banking union:

\[
E \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^H) + u^H(x_t, \theta) \right] \geq E \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^{H,0}) + u^H(x_t^0, \theta) \right],
\]

or

\[
E \sum_{t=s}^{\infty} \beta^{t-s} \left[ \pi v(r_{t,11}) + (1 - \pi) v(r_{t,01}) \right] \leq E \sum_{t=s}^{\infty} \beta^{t-s} \left[ \pi v(r_{t,10}) + (1 - \pi) v(r_{t,00}) \right].
\]

So

\[
E \sum_{t=s}^{\infty} \beta^{t-s} \pi \left( v(r_{t,10}) - v(r_{t,11}) + v(r_{t,01}) - v(r_{t,00}) \right) \geq E \sum_{t=s}^{\infty} \left[ v(r_{t,01}) - v(r_{t,00}) \right]
\]

Since \( v(r_{t,11}) < v(r_{t,10}) \) and \( v(r_{t,00}) < v(r_{t,01}) \), it follows that

\[
\pi^* = \frac{E \sum_{t=s}^{\infty} \left[ v(r_{t,01}) - v(r_{t,00}) \right]}{E \sum_{t=s}^{\infty} \beta^{t-s} \left( v(r_{t,10}) - v(r_{t,11}) + v(r_{t,01}) - v(r_{t,00}) \right)} < 1.
\]

### A.2.12 Proof of Proposition 9

The proof follows the same approach as the proof of Proposition 3 in Battaglini & Coate (2008). Let \( \psi_t(b') \) denote the distribution function of the current level of debt at the beginning of period \( t \). The distribution function \( \psi_1(b') \) is exogenous and determined by the initial level of debt \( b_0 \). Let \( \tilde{\Theta} = \{0, 1\} \times \Theta \), where the first set refers to the electoral shock and the second to the liquidity shock. Since the shocks are independent, let \( \tilde{P} \) denote the joint cumulative distribution over \( \tilde{\Theta} \).

The correspondence implied by the politician’s equilibrium choices and the
supranational authority’s equilibrium policy choices is given by \( T : [\bar{b}, \overline{\bar{b}}] \times [b, \bar{b}] \rightarrow \tilde{\Theta} : \)

\[
T(b, b') = \begin{cases} 
(0, \theta^0) & \text{if } b' < b'^\min(b) \\
\min\{ (\chi, \theta) \in \tilde{\Theta} : b'(b, \chi, \theta, \tau(b, \chi, \theta), x(b, \chi, \theta)) = b' \} & \text{if } b' \in [b'^\min(b), b'^\max(b)] \\
(1, \theta^N) & \text{if } b' > b'^\max(b) 
\end{cases}
\]

where

\[
b'^\min(b) = b'(b, 0, \theta^0, \tau(b, 0, \theta^0), x(b, 0, \theta^0)),
\]

\[
b'^\max(b) = b'(b, 1, \theta^N, \tau(b, 1, \theta^N), x(b, 1, \theta^N)).
\]

The correspondence \( T(b, b') \) gives the minimum combination of shocks under which the equilibrium new debt level would be \( b' \), given outstanding debt \( b \). Then, the transition function is given by

\[
H(b, b') = \tilde{P}(T(b, b')).
\]

The function \( H(b, b') \) gives the probability that next period’s debt will be less than or equal to \( b' \) given the current outstanding debt \( b \). Then, the distribution of debt at the beginning of any period \( t \geq 2 \) is defined inductively by

\[
\psi_t(b') = \int_{\bar{b}} H(b, b') d\psi_{t-1}(b).
\]

The sequence of distributions \( \psi_t(b') \) converges to distribution \( \psi(b') \) if \( \forall b \in [\bar{b}, \overline{\bar{b}}] \),

\[
\lim_{t \rightarrow \infty} \psi_t(b') = \psi(b').
\]

The limiting distribution is invariant if \( \psi^*(b') = \int_{\bar{b}} H(b, b') d\psi^*(b) \).

To prove that the sequence of distributions converges to a unique invariant distribution, we must first prove that \( H(b, b') \) has the Feller Property, and that it is monotonic in \( b \). By Theorem 12.12 in Lucas, Stokey & Prescott (1989), the following mixing condition must be satisfied: \( \exists \epsilon > 0 \) and
m \geq 1$, such that for any $b^* \in (\bar{b}, \tilde{b})$, $H^m(\tilde{b}, b^*) \geq \epsilon$ and $1 - H^m(\bar{b}, b^*) \geq \epsilon$, where the function $H^m(b, b')$ is defined inductively by $H^1(b, b') = H(b, b')$, and $H^m(b, b') = \int_z H(z, b')dH^{m-1}(b, z)$. This condition requires that starting from the highest level of debt $\tilde{b}$, we will end up at or below debt $b^*$ with probability greater than $\epsilon$ after $m$ periods, and if we start with the lowest level of debt, we will end up at or above $b^*$ with probability greater than $\epsilon$ in $m$ periods.

We use the monotonicity properties of the equilibrium policy functions, with respect to both $b$ and the shocks $\theta$ and $\chi$ to show that the mixing condition is satisfied.

For any $b \in [\bar{b}, \tilde{b}]$ and $(\chi, \theta) \in \tilde{\Theta}$ define the sequence $\langle \phi_m(b, \chi, \theta) \rangle$ as follows: $\phi_0(b, \chi, \theta) = b$, $\phi_{m+1}(b, \chi, \theta) = b'(\phi_m(b, \chi, \theta), \chi, \theta)$, assuming that the supranational authority is following the equilibrium policies $\tau(b, \chi, \theta)$ and $\chi(b, \chi, \theta)$.

This means that $\phi_m(b, \chi, \theta)$ is the level of new debt starting from outstanding debt $b$, and assuming the same pair of shocks $(\chi, \theta)$ is repeated in periods 1 through $m$. By the setup of the model, there is a positive probability on each pair $(\chi, \theta)$, therefore $\hat{P}(\chi', \theta') - \hat{P}(\chi, \theta) > 0$ for $\theta' > \theta$. This implies that, for a small $\lambda^m$, $H^m(b, \phi_m(b, 0, \theta^0)) - H^m(b, \phi_m(b, 0, \theta^0)) = (\xi \lambda^m)^{m-1} > 0$.

Using the above, it can be shown that $H^m(\tilde{b}, b^*) > 0$, for $m$ sufficiently large. It suffices to show that, for $m$ sufficiently large, $T(\phi_m(\bar{b}, 0, \theta^0), b^*) > (0, \theta^0)$. Then, for any such $m$, by continuity, there exits a small $\lambda^m$ such that $T(\phi_m(\tilde{b}, 0, \theta^0 + \lambda^m), b^*) > (0, \theta^0)$. So,

$$H^m(\tilde{b}, b^*) = \int_z H(z, b^*)dH^{m-1}(\tilde{b}, z) = \int_z \hat{P}(T(z, b^*))dH^{m-1}(\tilde{b}, z)$$

$$\geq \int_{\phi_m(\tilde{b}, 0, \theta^0)} \hat{P}(T(z, b^*))dH^{m-1}(\tilde{b}, z)$$

$$\geq \hat{P}(T(\phi_m(\tilde{b}, 0, \theta^0 + \lambda^m), b^*)) \left[ H^{m-1}(\tilde{b}, \phi_{m-1}(\tilde{b}, 0, \theta^0 + \lambda^m)) - H^{m-1}(\tilde{b}, \phi_{m-1}(\tilde{b}, 0, \theta^0)) \right]$$

$$\geq \hat{P}(T(\phi_m(\tilde{b}, 0, \theta^0 + \lambda^m), b^*))(\xi \lambda^m)^{m-1} > 0.$$

Suppose, to the contrary, that $T(\phi_m(\bar{b}, 0, \theta^0), b^*) \leq (0, \theta^0)$. Then, from the politician’s first-order conditions, the realization of shocks $(0, \theta^0)$ implies that
we obtain a decreasing sequence \( \{ \phi_m(\bar{b}, 0, \theta^0) \}_m \). Suppose that \( \phi_m(\bar{b}, 0, \theta^0) \) converged to some \( b^{**} > \bar{b} \). Then, in the limit, by the continuity of the policy functions, \( \lim_{m \to \infty} g(\phi_m(\bar{b}, 0, \theta^0), \chi, \theta) = g(b^{\infty}, \chi, \theta) \), for all pairs \((\chi, \theta)\). However, the policy \( g \) is strictly decreasing in \( \theta \), and by (17) and (18) at \( \chi = 0 \), \( b' \) must be decreasing, which contradicts the convergence assumption. The analogous argument can be made starting from \( \bar{b} \), given repeated \( \theta^N \) shocks, to show that \( 1 - H^m(b, b^*) \geq \epsilon \). Thus, the necessary conditions are satisfied for a unique invariant distribution.